Incremental Construction of Minimal Finite State Automata

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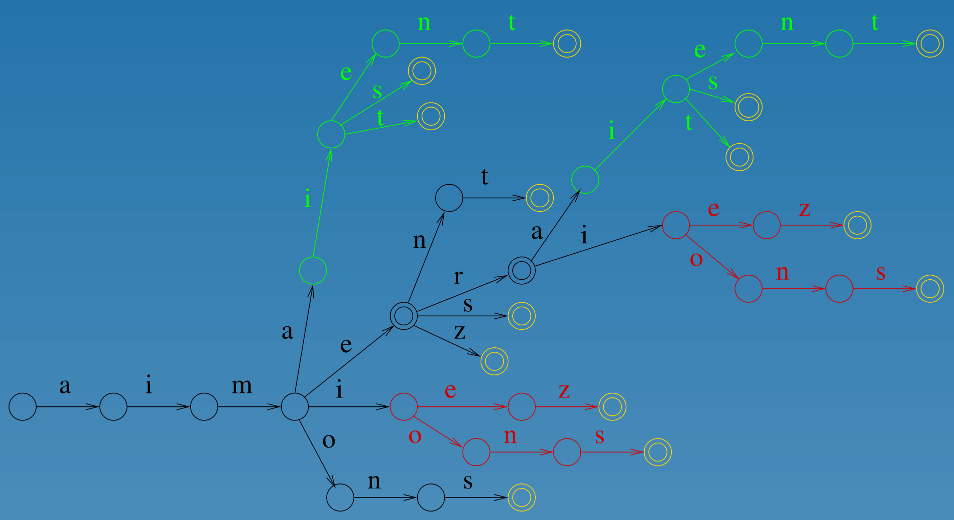
Overview

- Incrementality and semi-incrementality
- Trie construction, minimization, synchronization
- Incremental algorithm for sorted data
- Unsorted data and confluence states
- Incremental algorithm for unsorted data
- Performance

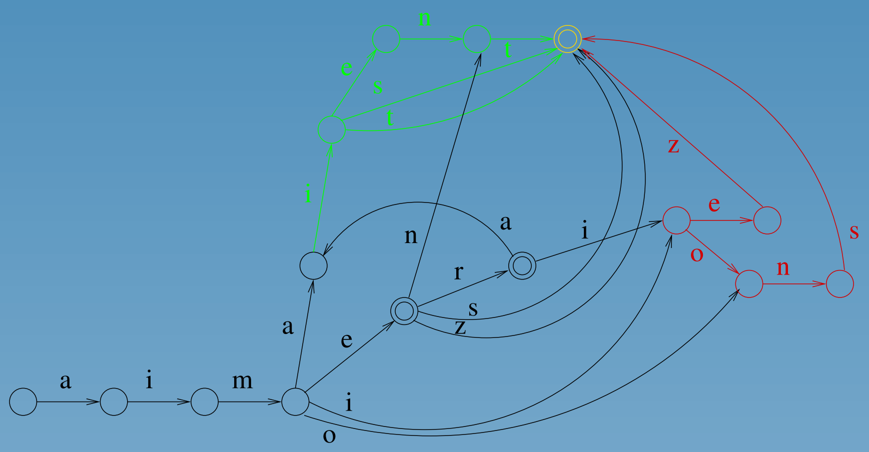
Incrementality and automata

- Final automata
  - ◇ ideal implementation of dictionaries
  - ◇ very efficient once constructed
  - ◇ traditional construction needs much memory
- Incremental and semi-incremental construction requires less memory
- Moore's law

Traditional construction

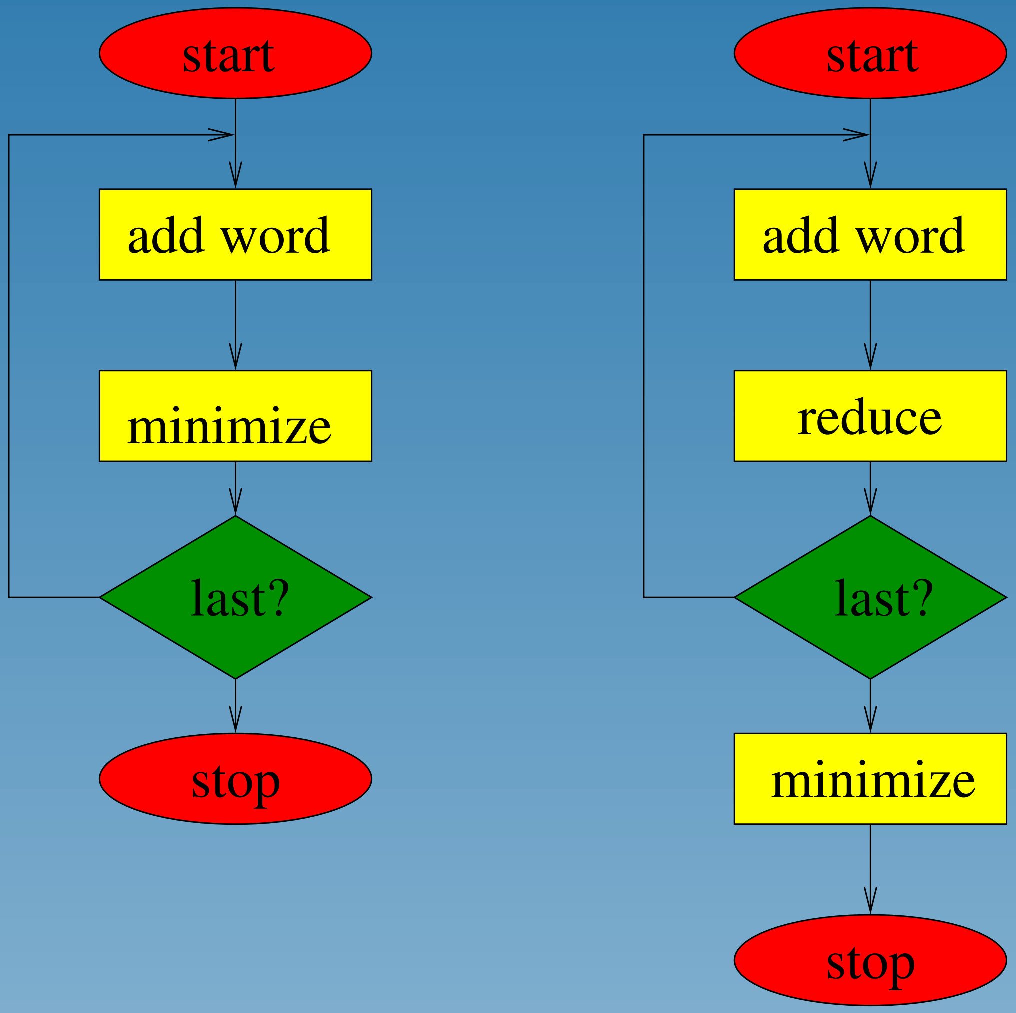


- Construct a trie



- Minimize it

Incremental vs. semi-incremental construction algorithms



Incremental and semi-incremental algorithms for acyclic automata

- Incremental algorithm for (lexicographically) sorted data (Daciuk, Mihov, Ciura, Deorowicz)

- Incremental algorithm for unsorted data (Aoe, Morimoto, Hase, Sgarbas, Fakotakis, Kokkinakis, Daciuk, Watson, Revuz. . .)

- Semi-incremental algorithm for data lexicographically sorted on reversed strings (Revuz)

- Semi-incremental algorithm for data sorted on decreasing length of strings (Watson)

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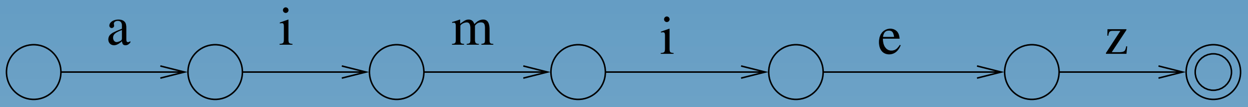

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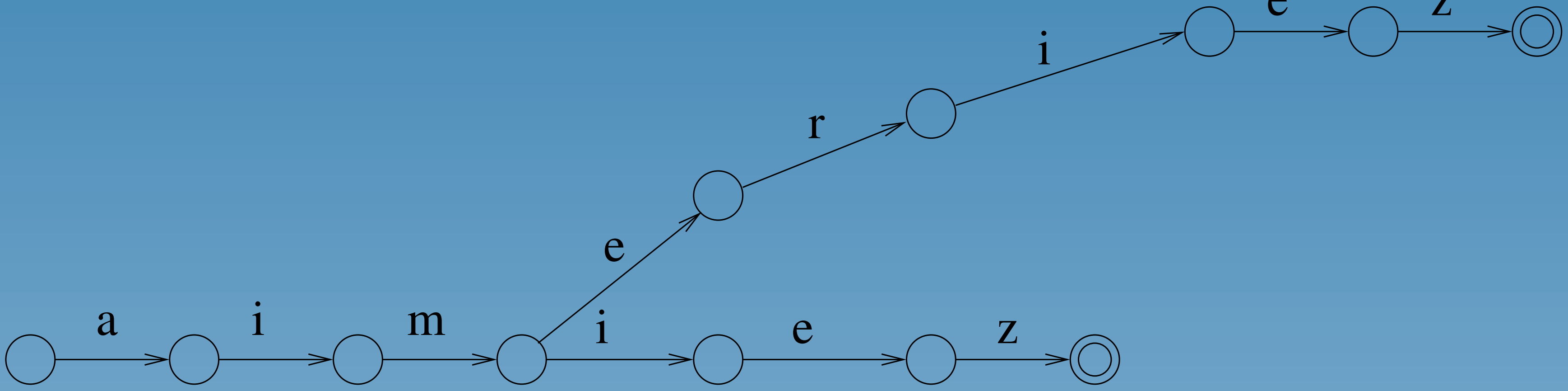

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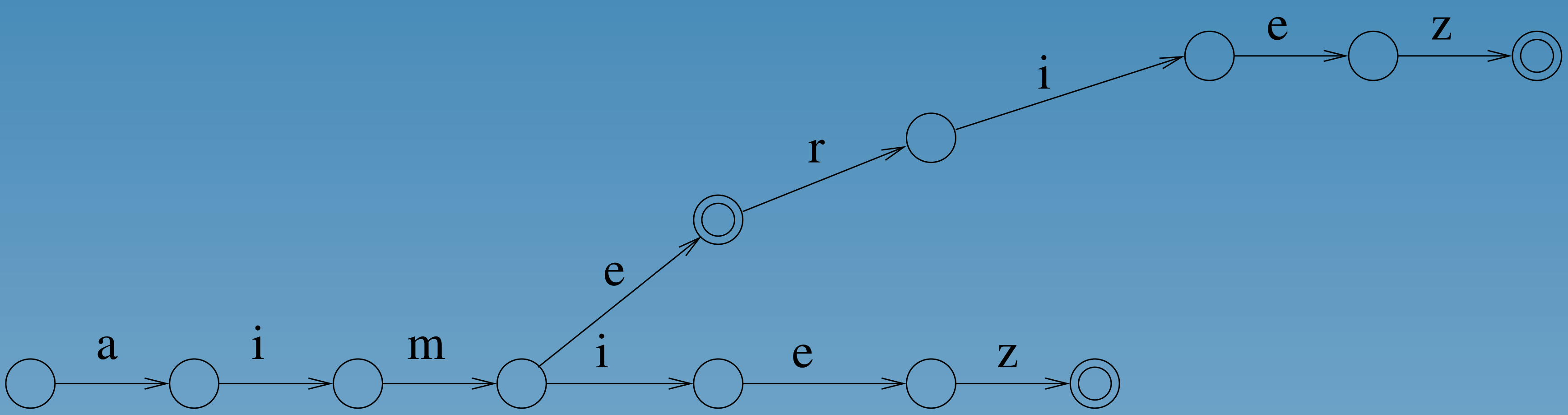

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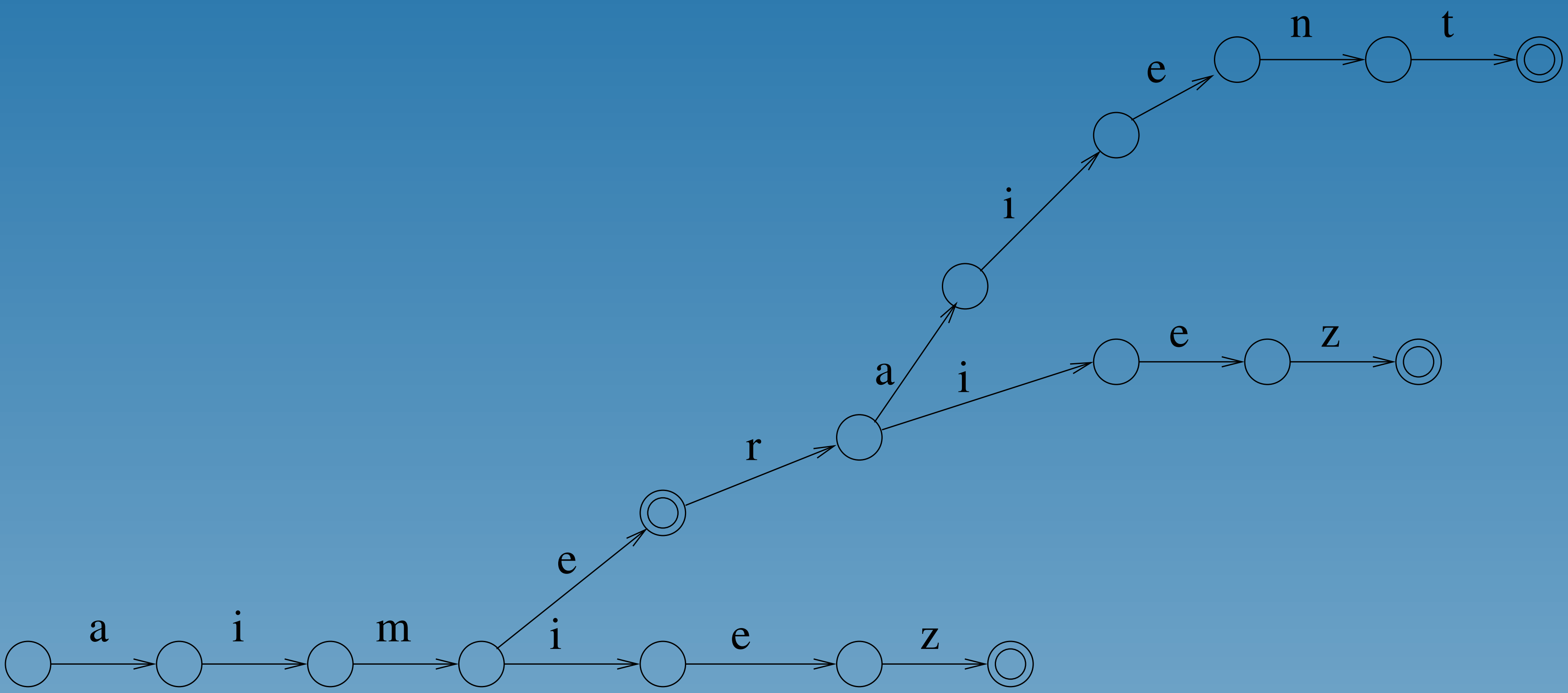

What is actually minimization?

- $M = (Q, \Sigma, \delta, q_0, F)$, $|M| = |Q|$
- M is minimal iff $\forall_{M' : \mathcal{L}(M') = \mathcal{L}(M)} |M| < |M'|$
- $\overrightarrow{\mathcal{L}}(q) = \{w : \delta^*(q, w) \in F\}$, $\mathcal{L}(M) = \overrightarrow{\mathcal{L}}(q_0)$
- $p \equiv q$ iff $\overrightarrow{\mathcal{L}}(p) = \overrightarrow{\mathcal{L}}(q)$.
- M is minimal iff $\forall_{p,q \in Q}\, p \equiv q \Leftrightarrow p = q$
- $\overrightarrow{\mathcal{L}}(q) = \cup_{a:\delta(q,a) \neq \perp}\; a\overrightarrow{\mathcal{L}}(\delta(q, a)) \cup \begin{cases} \emptyset & q \notin F \\ \epsilon & q \in F \end{cases}$

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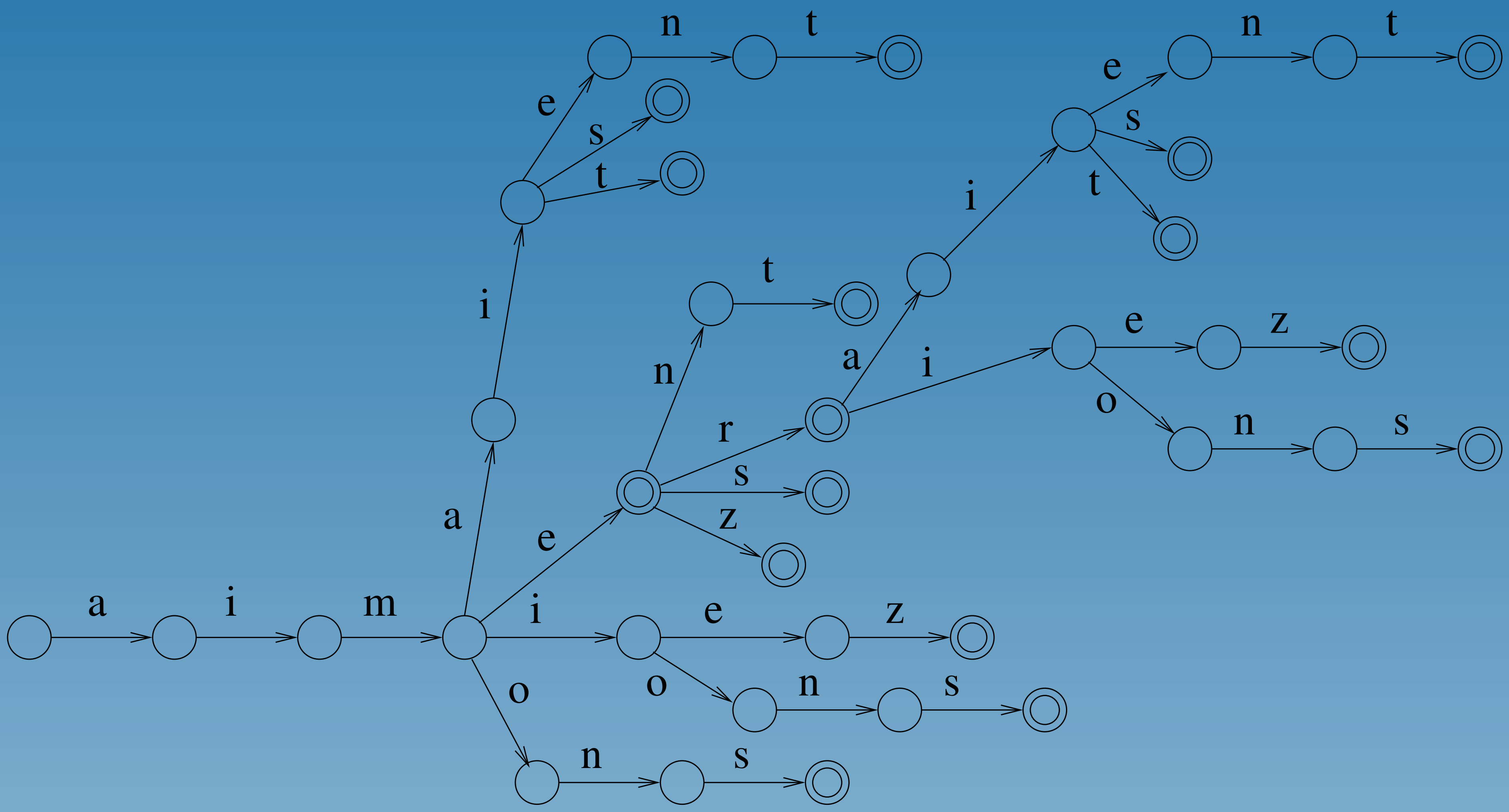

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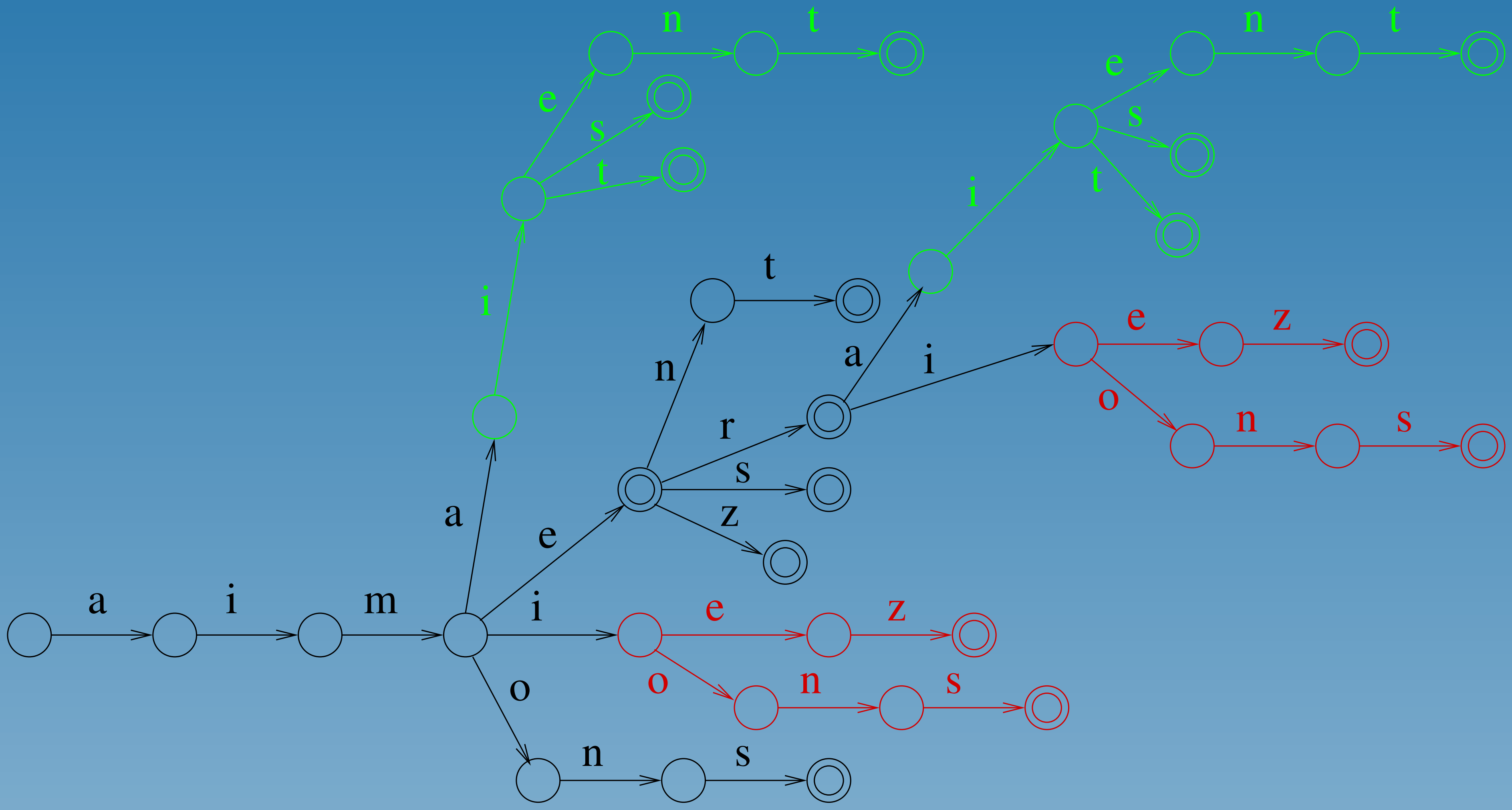

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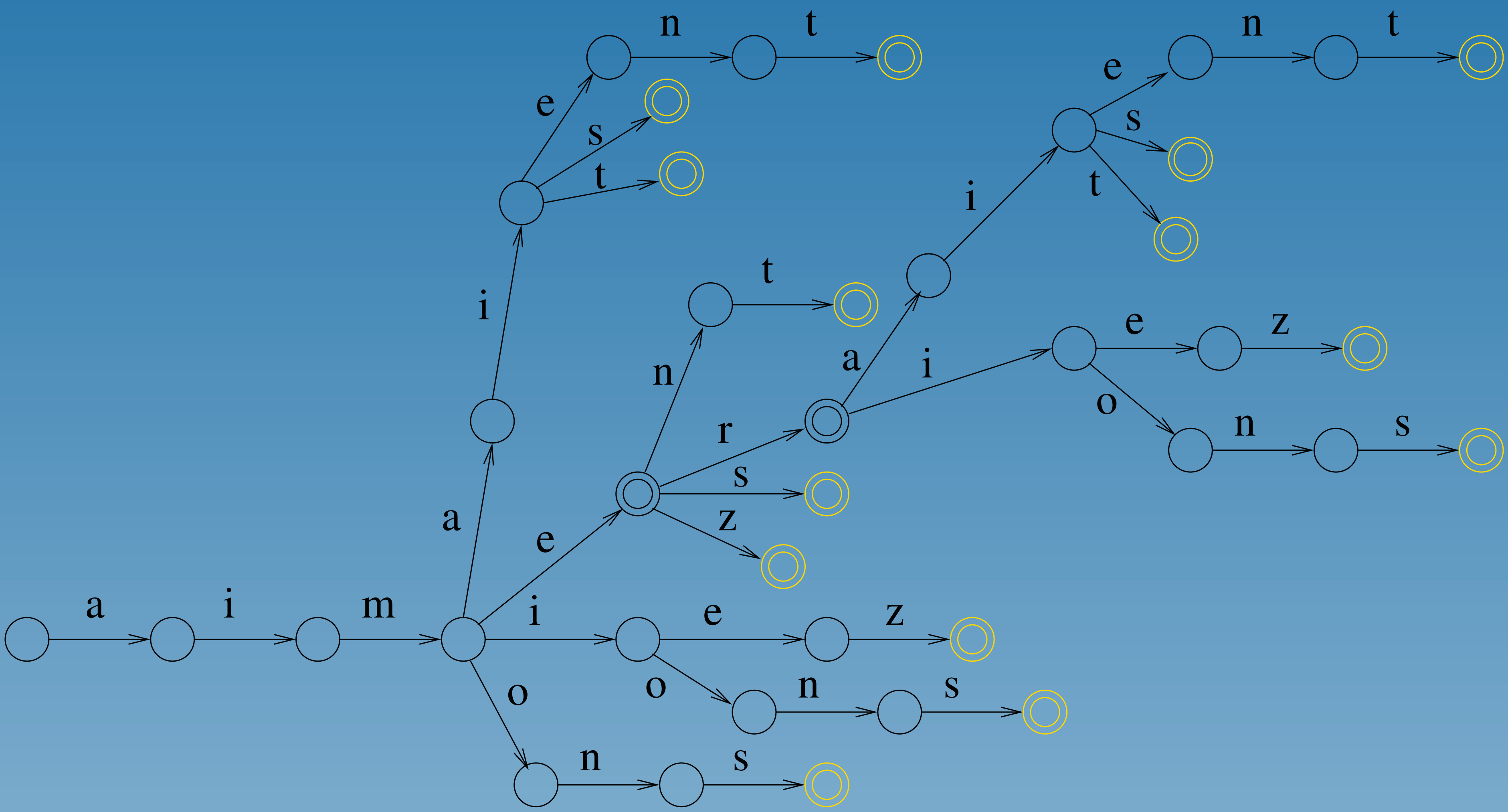

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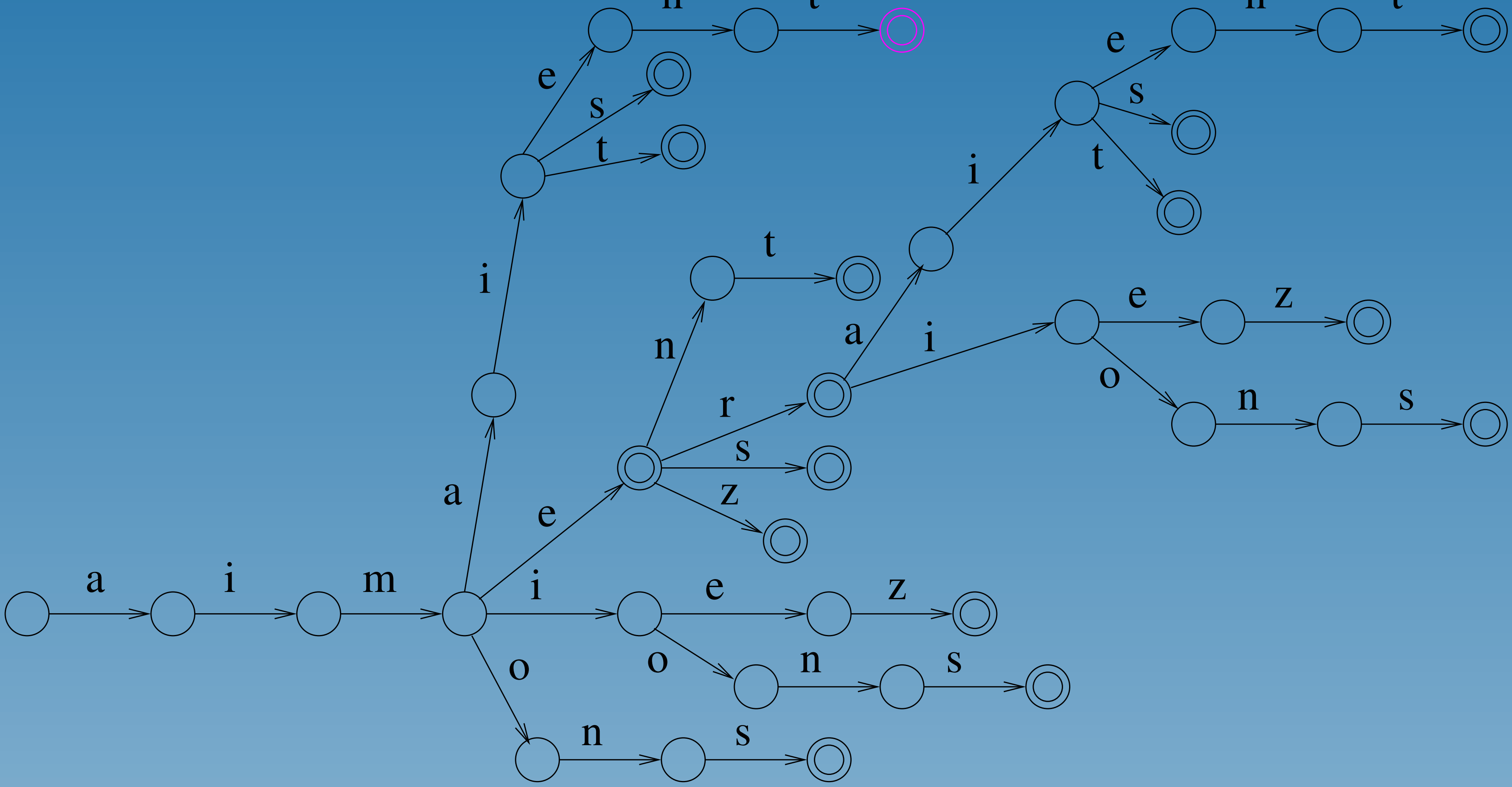

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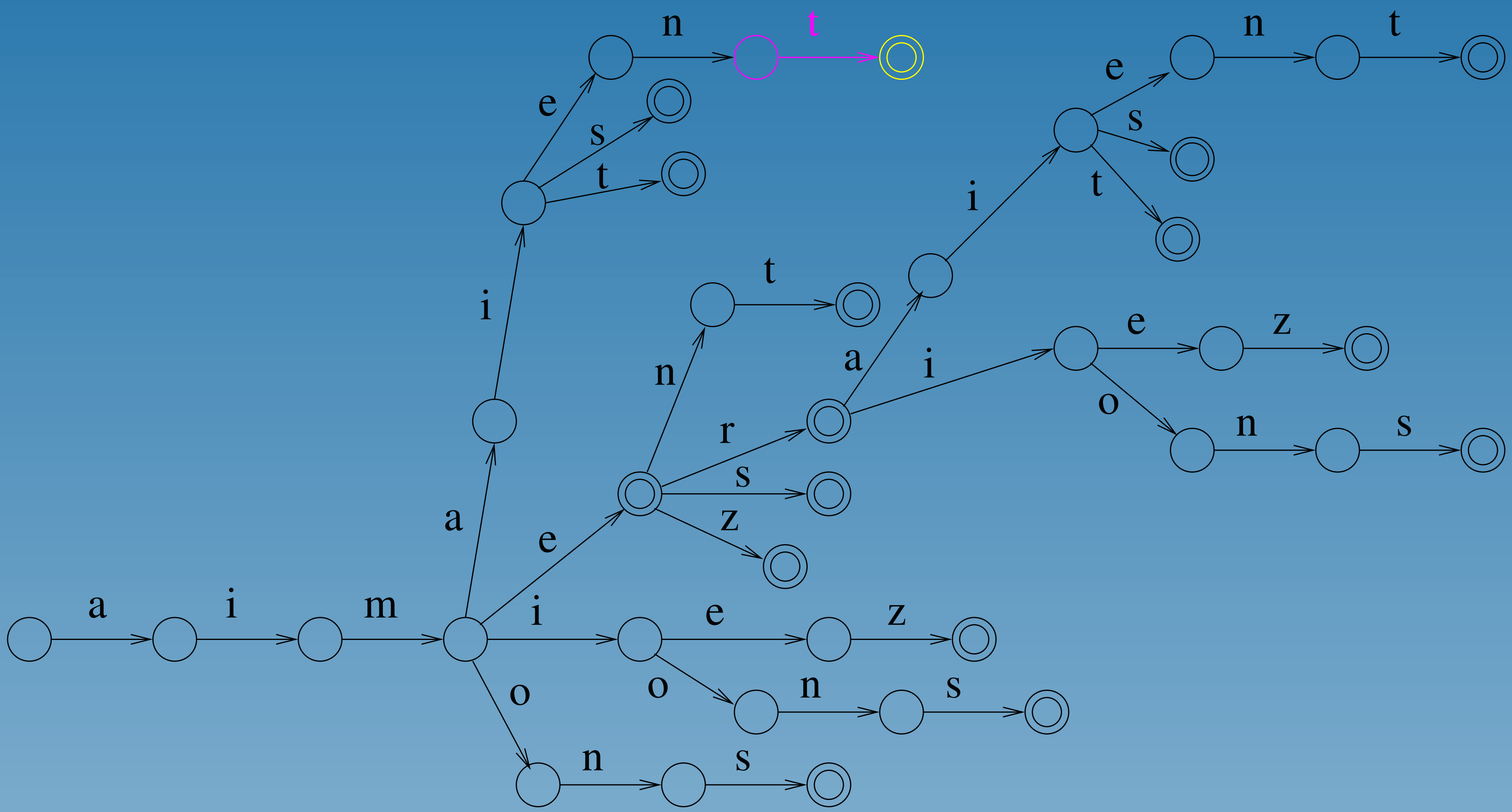

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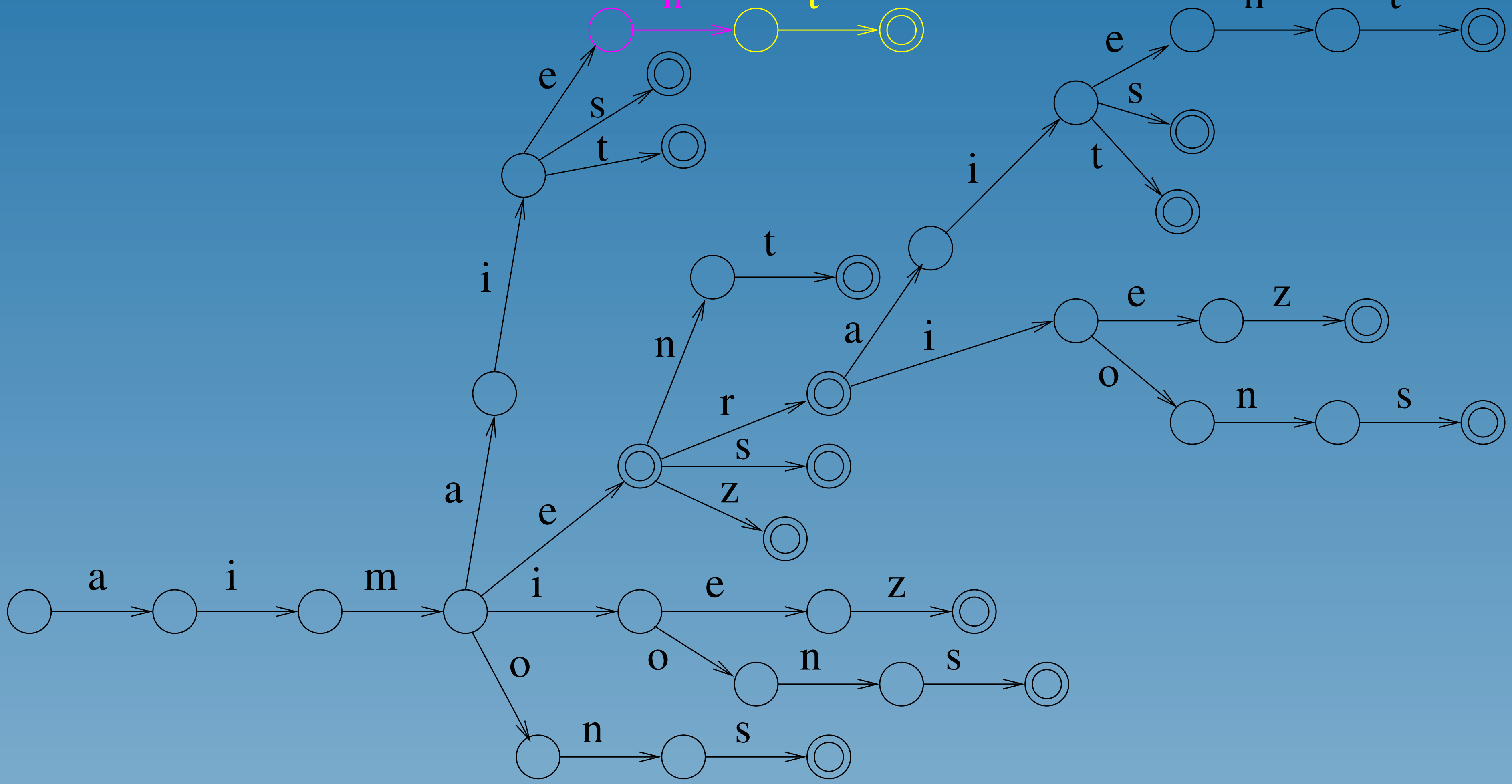

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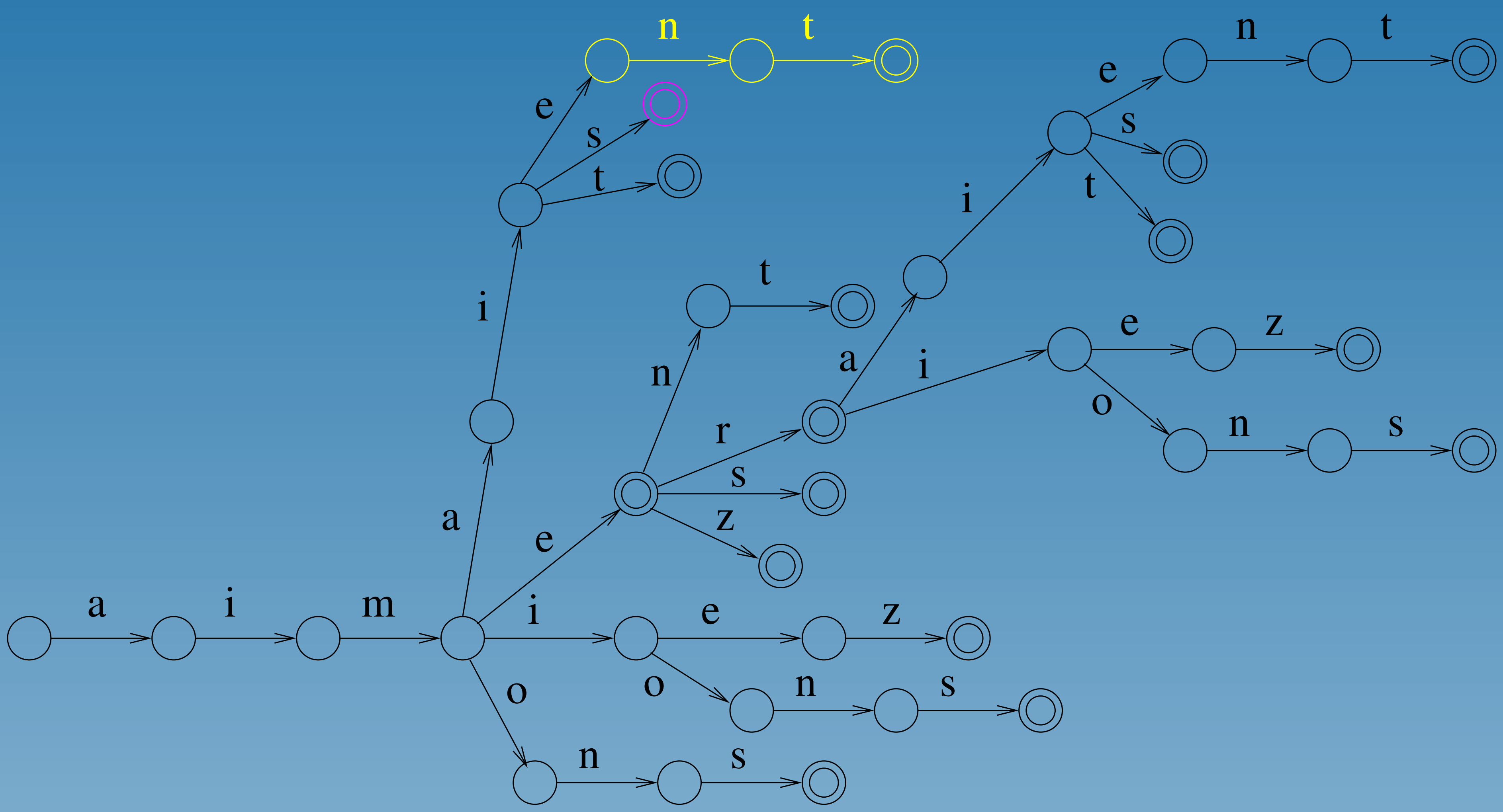

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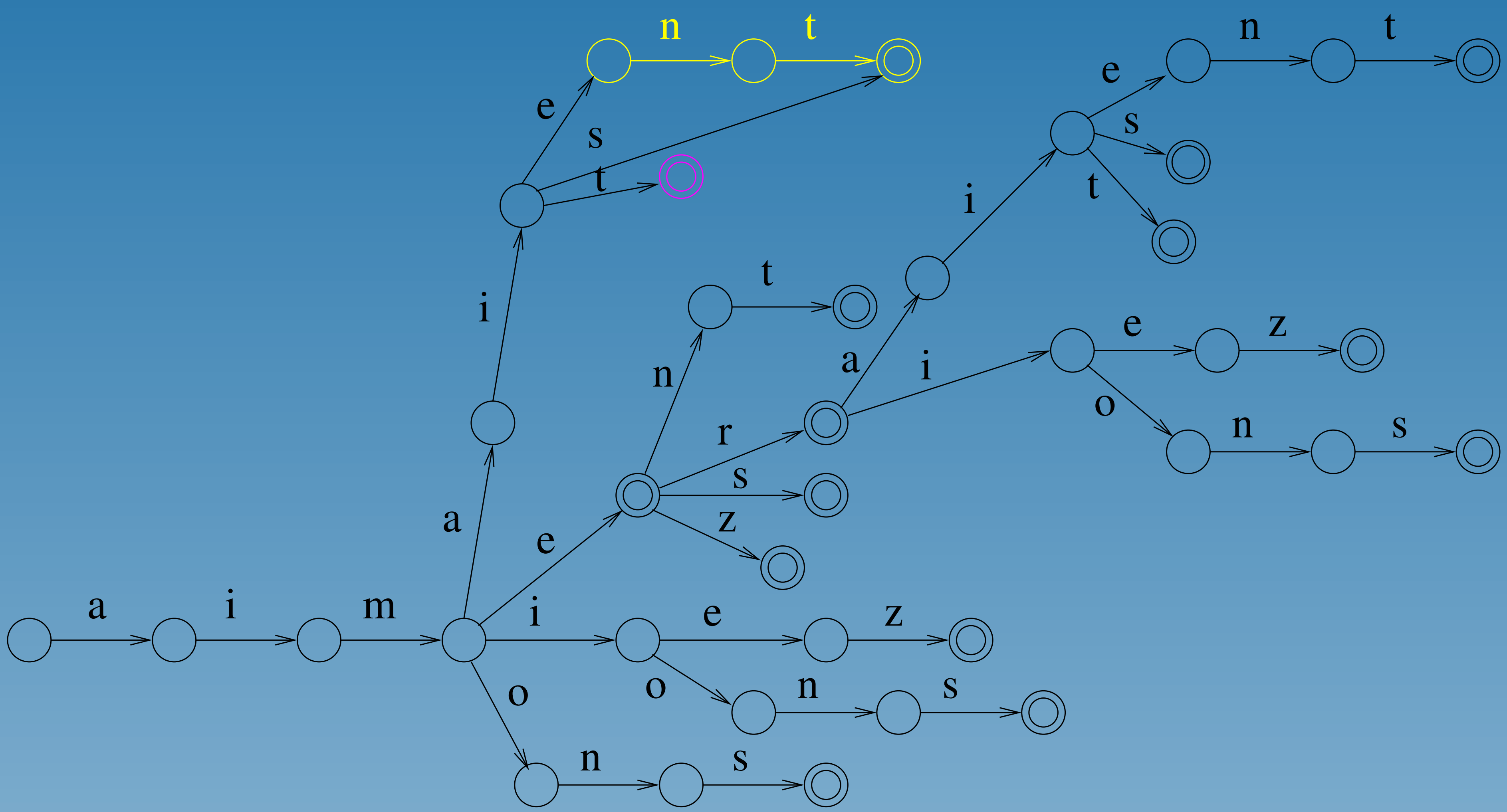

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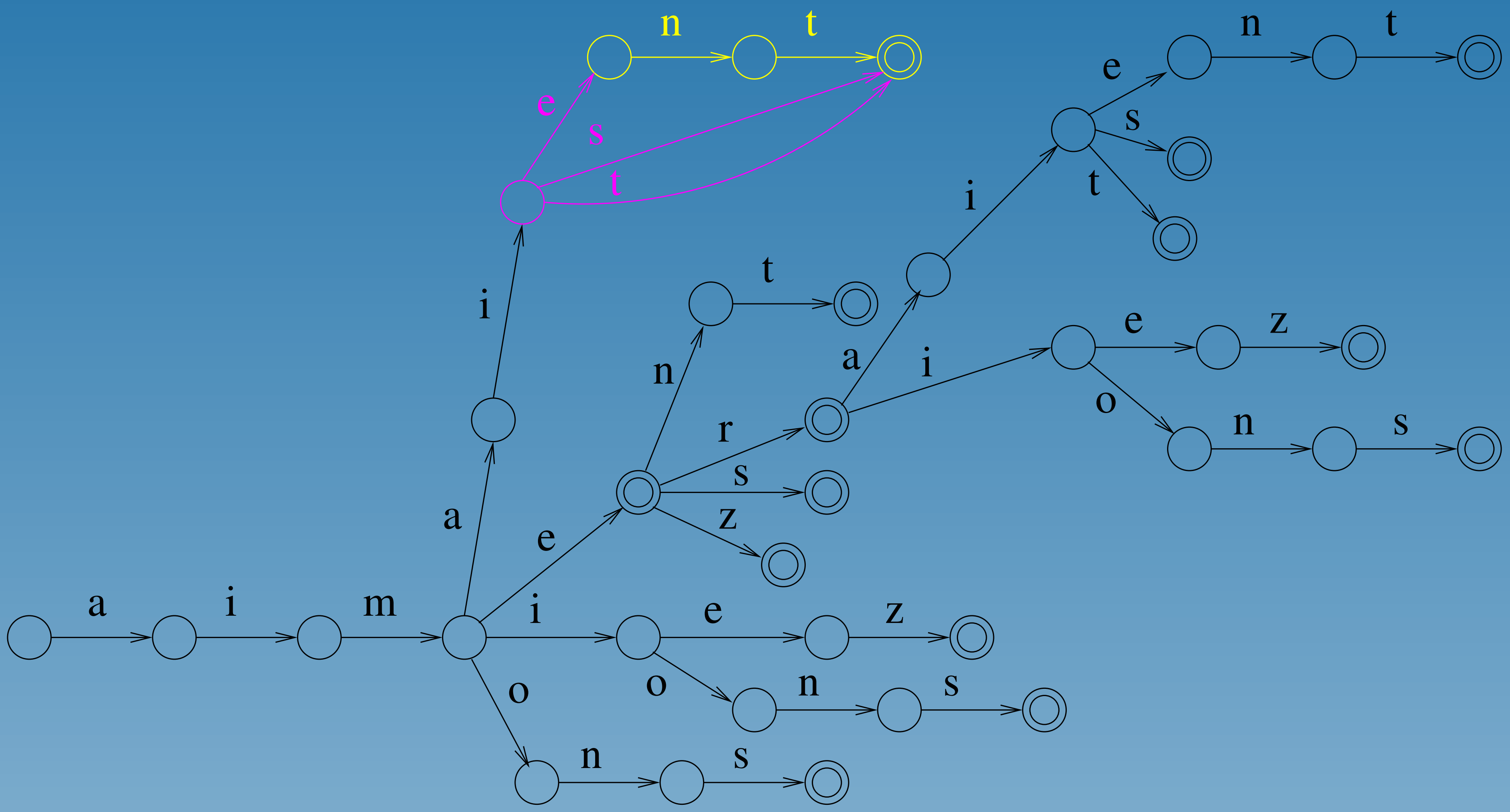

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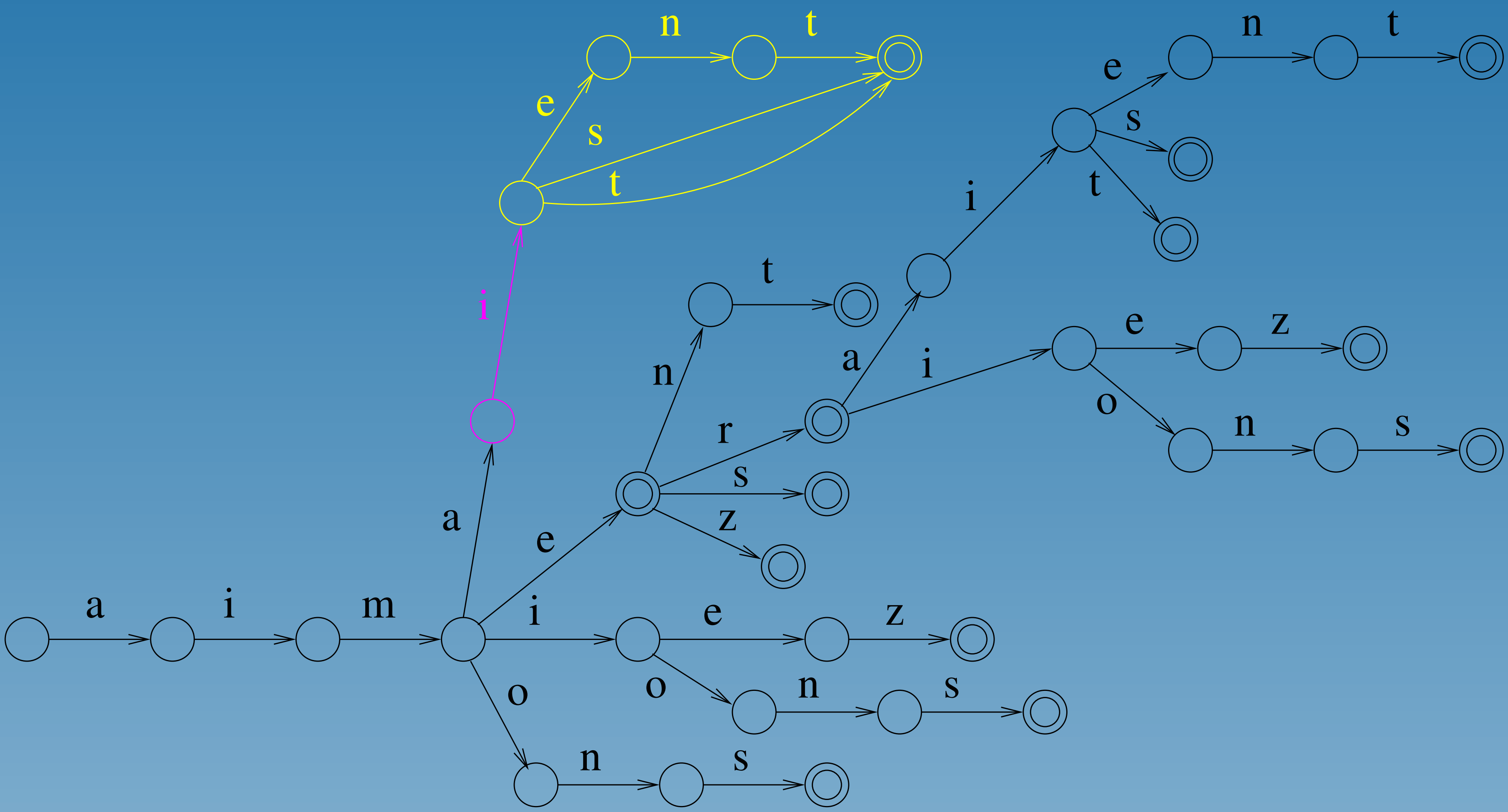

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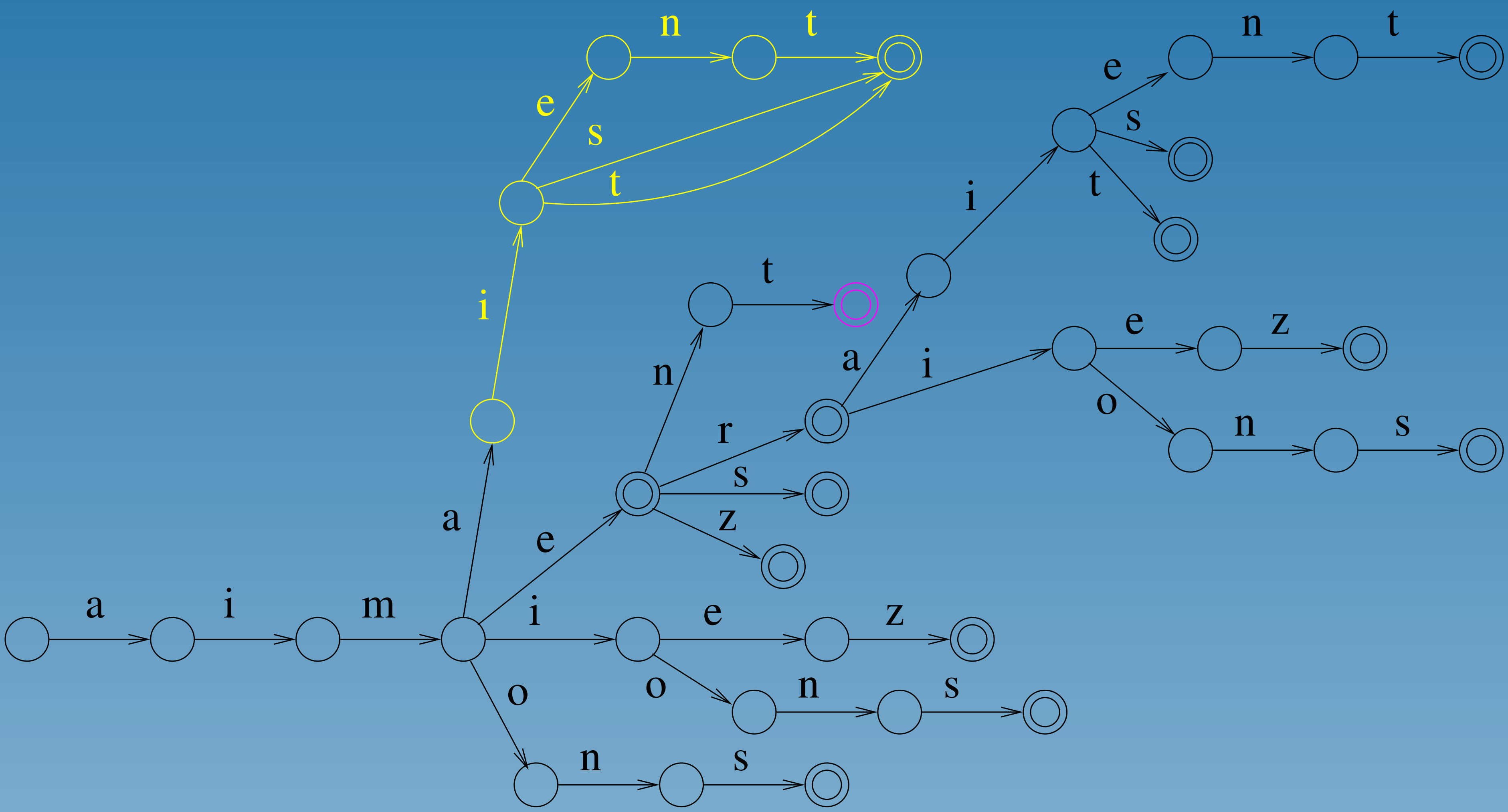

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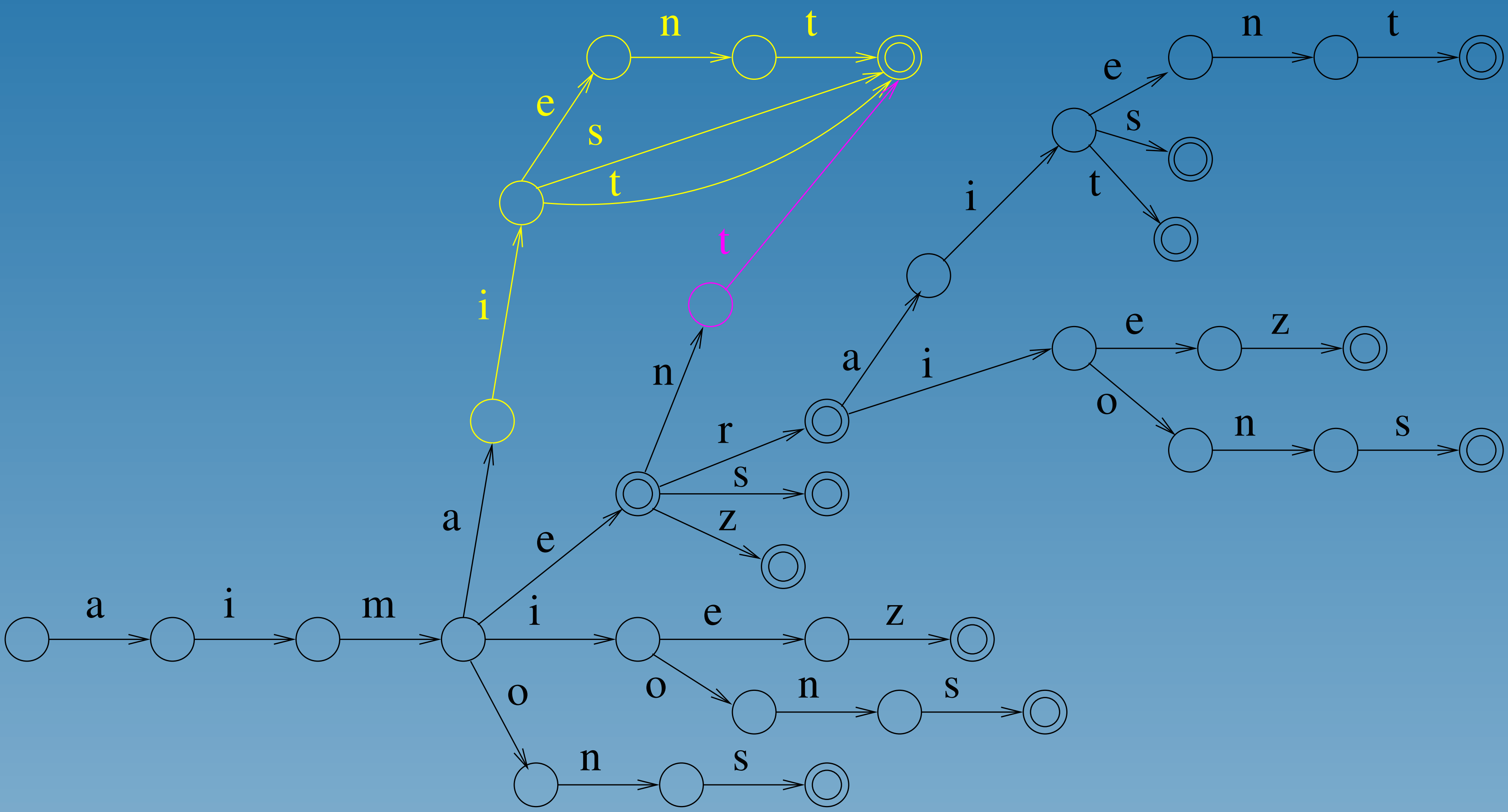

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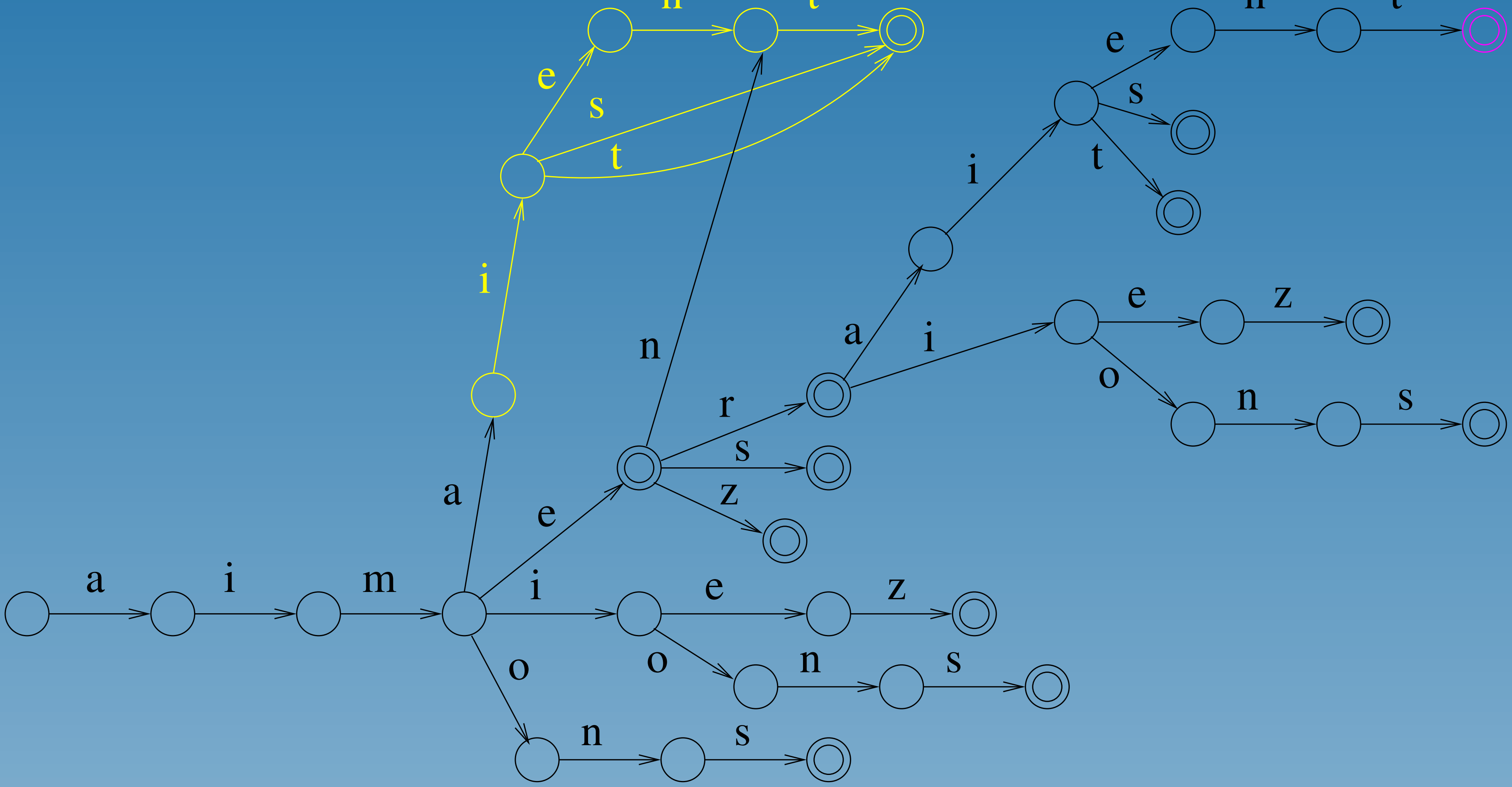

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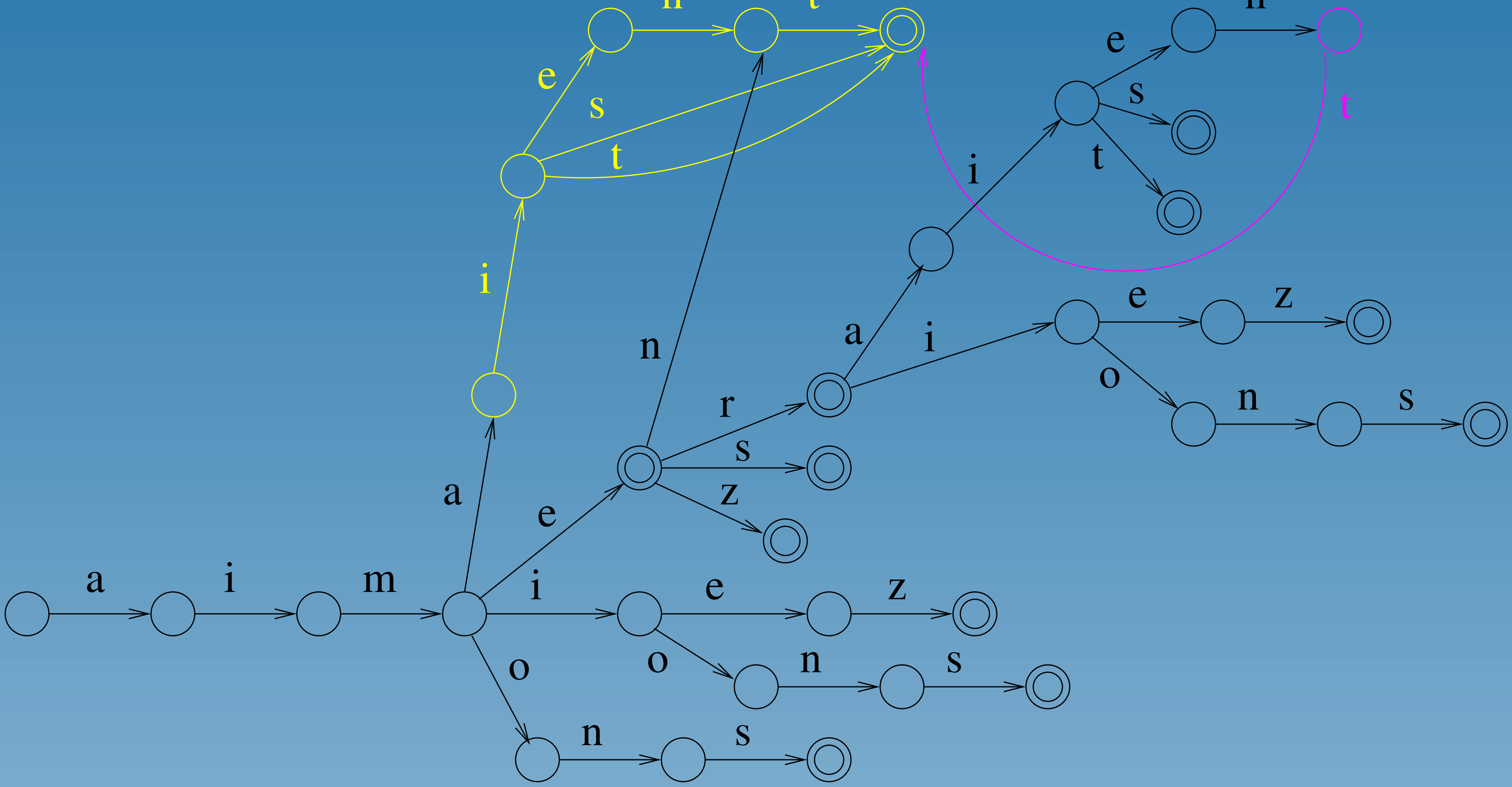

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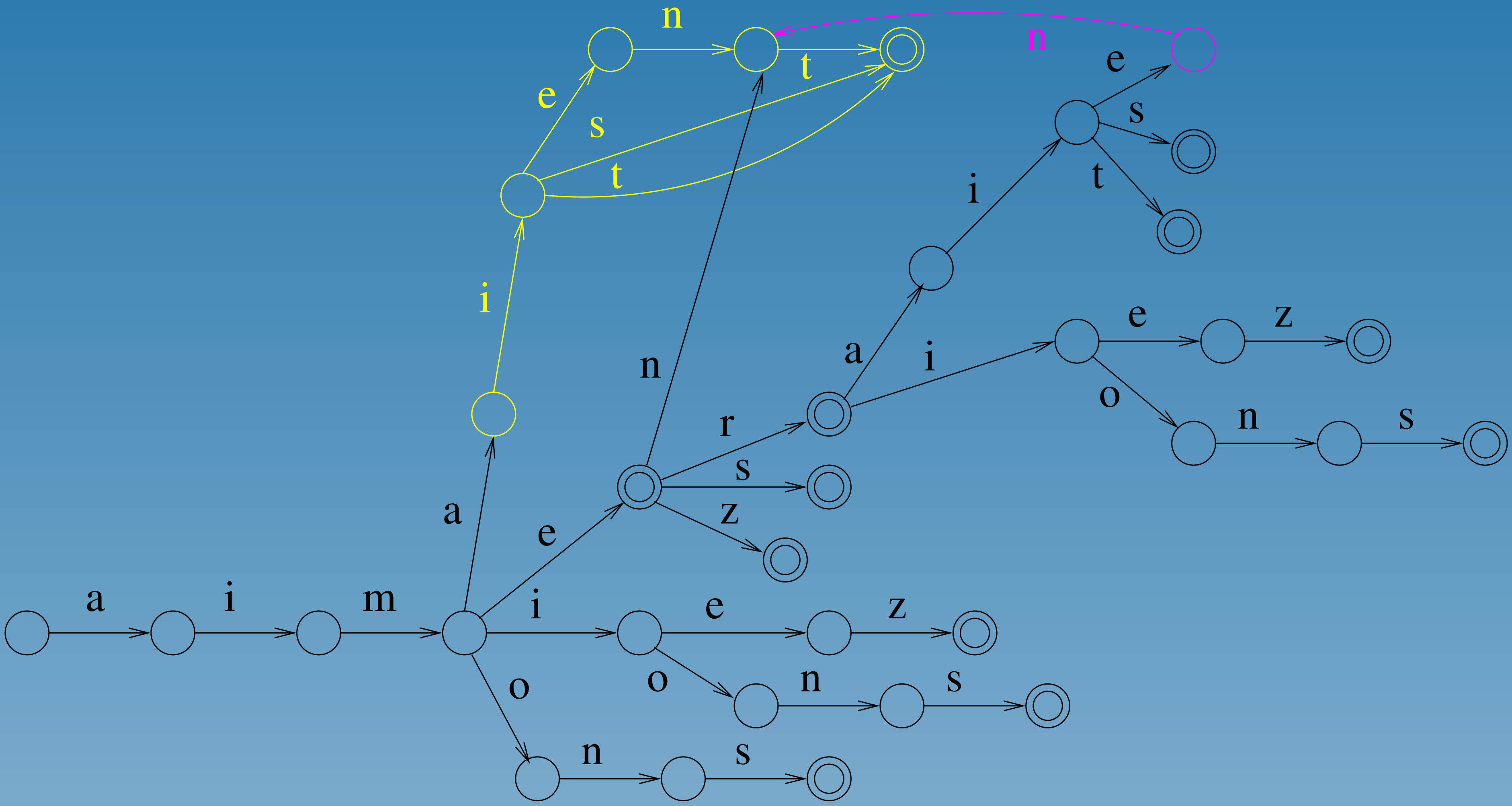

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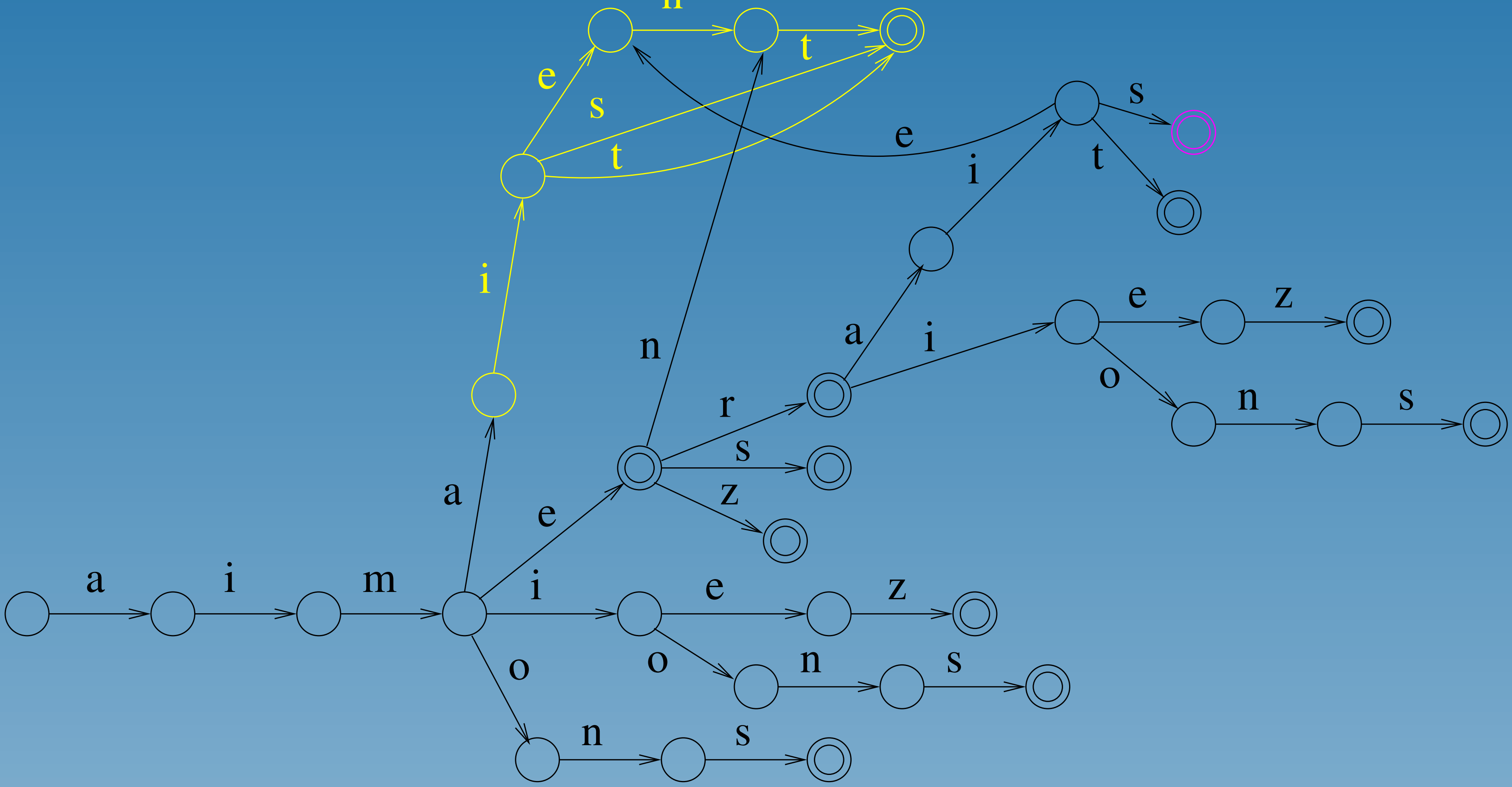

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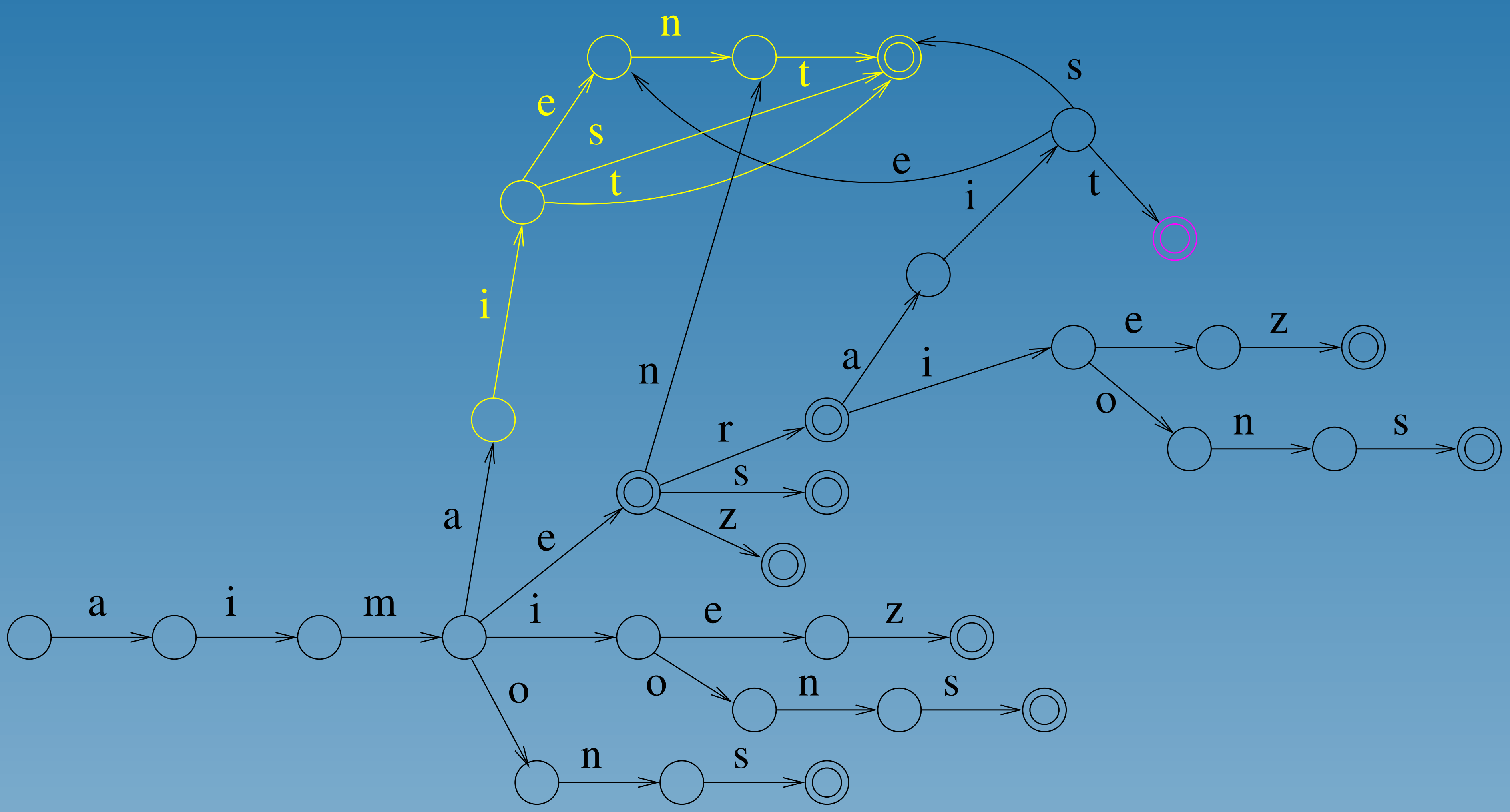

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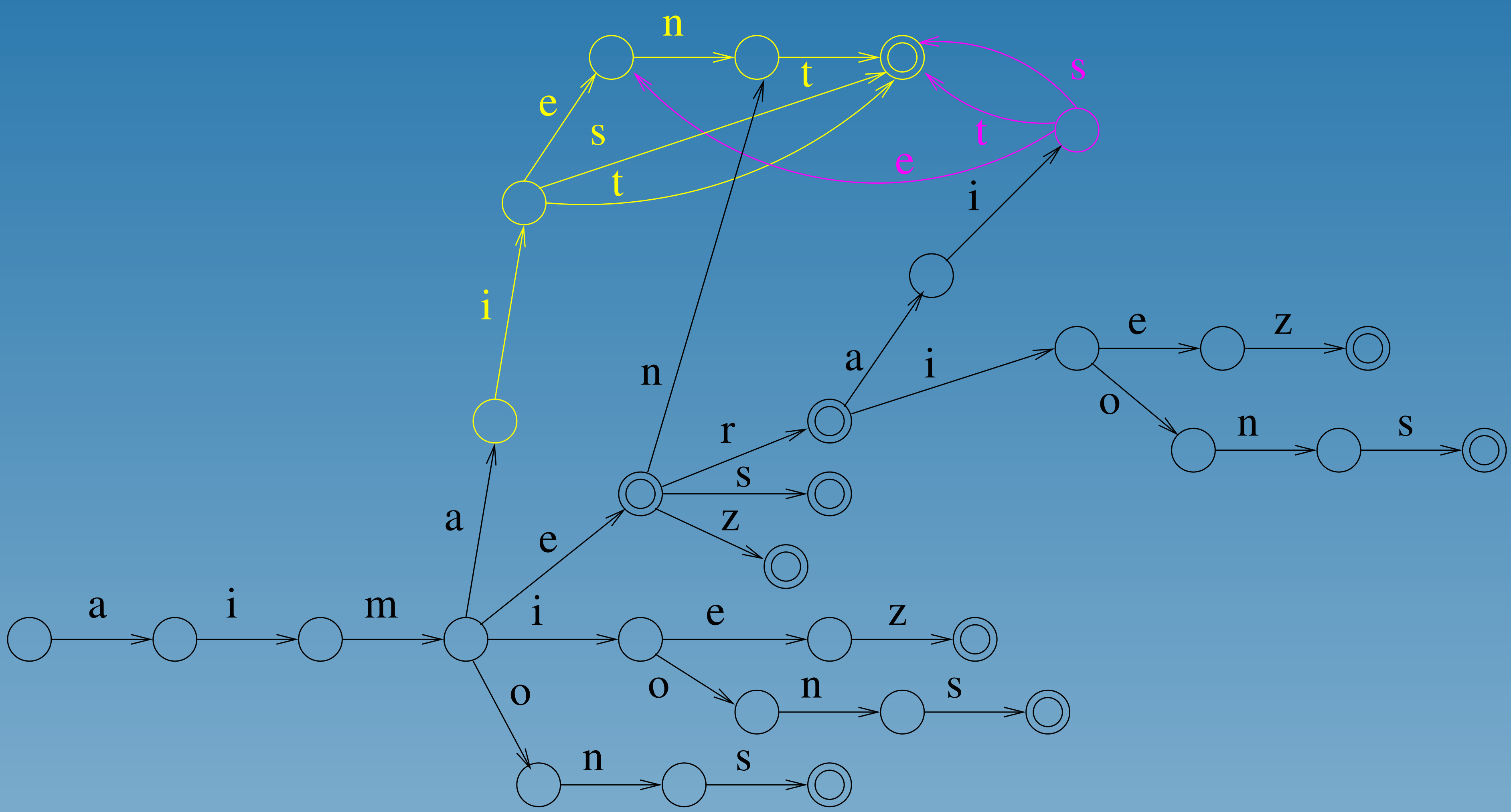

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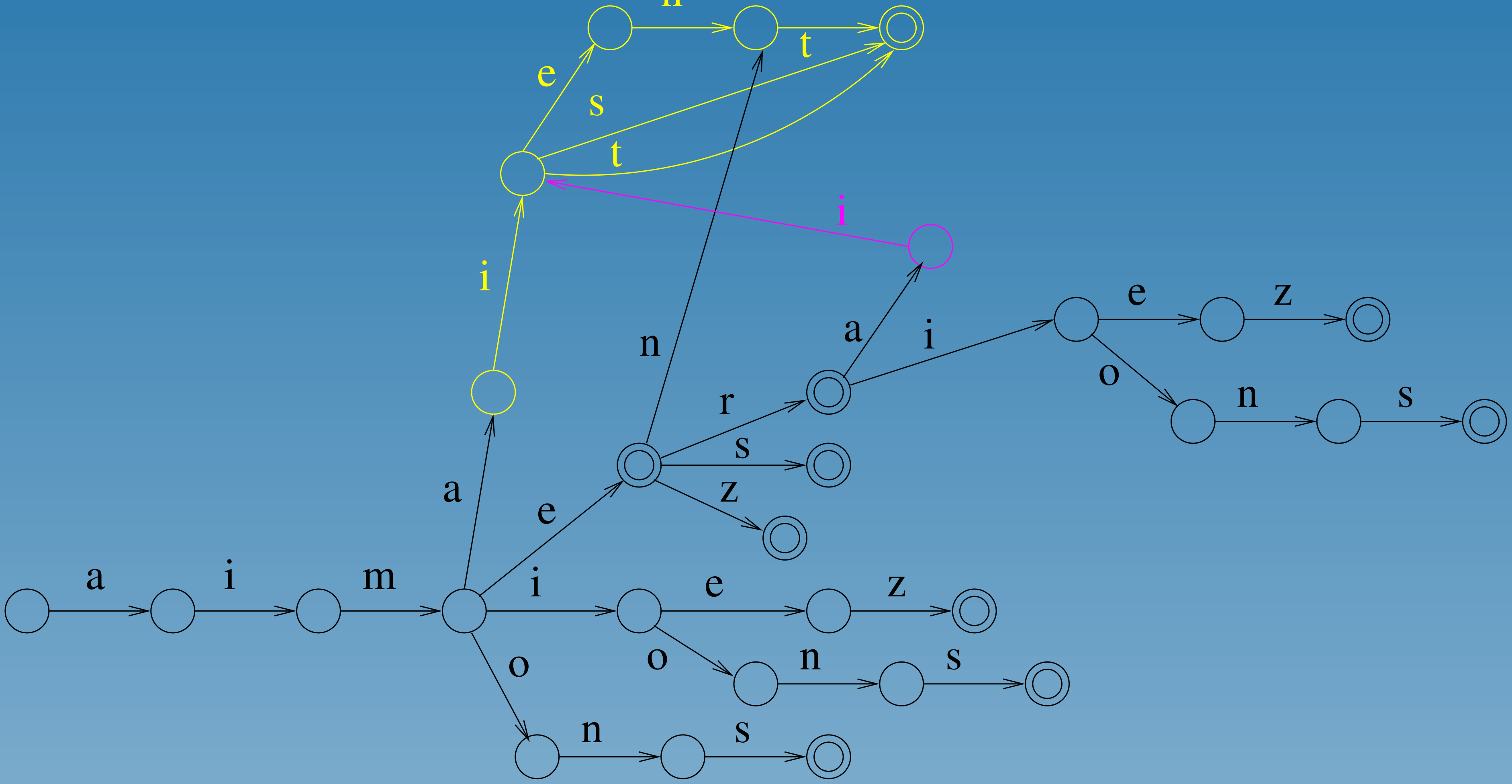

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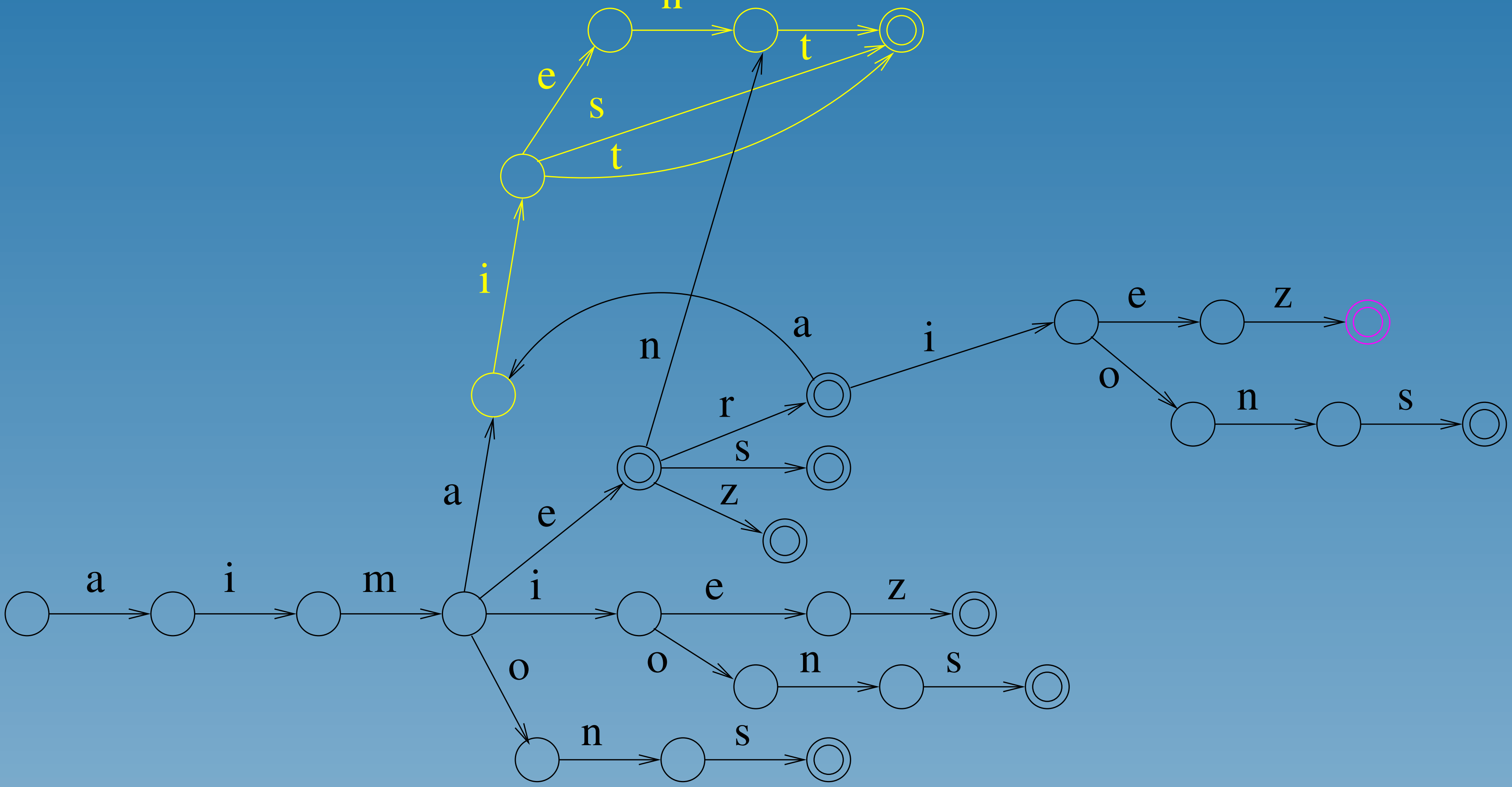


The register

How does one check equivalence of two states?

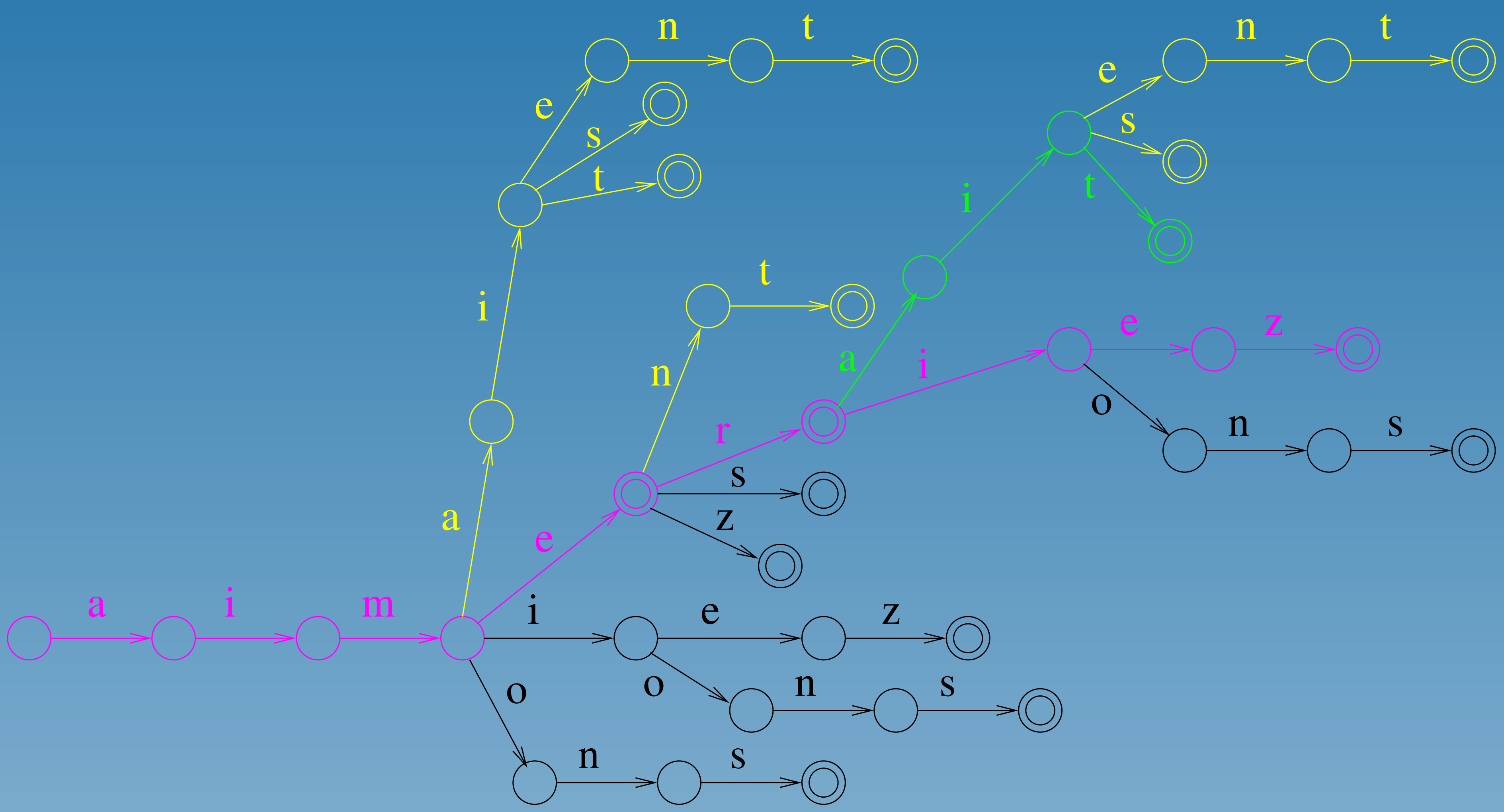
- Use the recursive definition of $\overrightarrow{\mathcal{L}}(q)$
- Visit states using postorder, so that children have unique right languages
- Keep pointers to states with unique $\overrightarrow{\mathcal{L}}(q)$ in a sparse table
- Use hash function on finality and transitions

Result: Operations on the register are $\mathcal{O}(1)$

Synchronization

- Add a word to the language of the automaton and minimize the whole automaton again
  - ◇ does not pose any restrictions on input data
  - ◇ the same states have to processed over and over again – slow
- Add a word to the language of the automaton and minimize the part that will not change in the future
  - ◇ requires data to be sorted in some way
  - ◇ faster due to procesing of states only when once

Synchronization – sorted data



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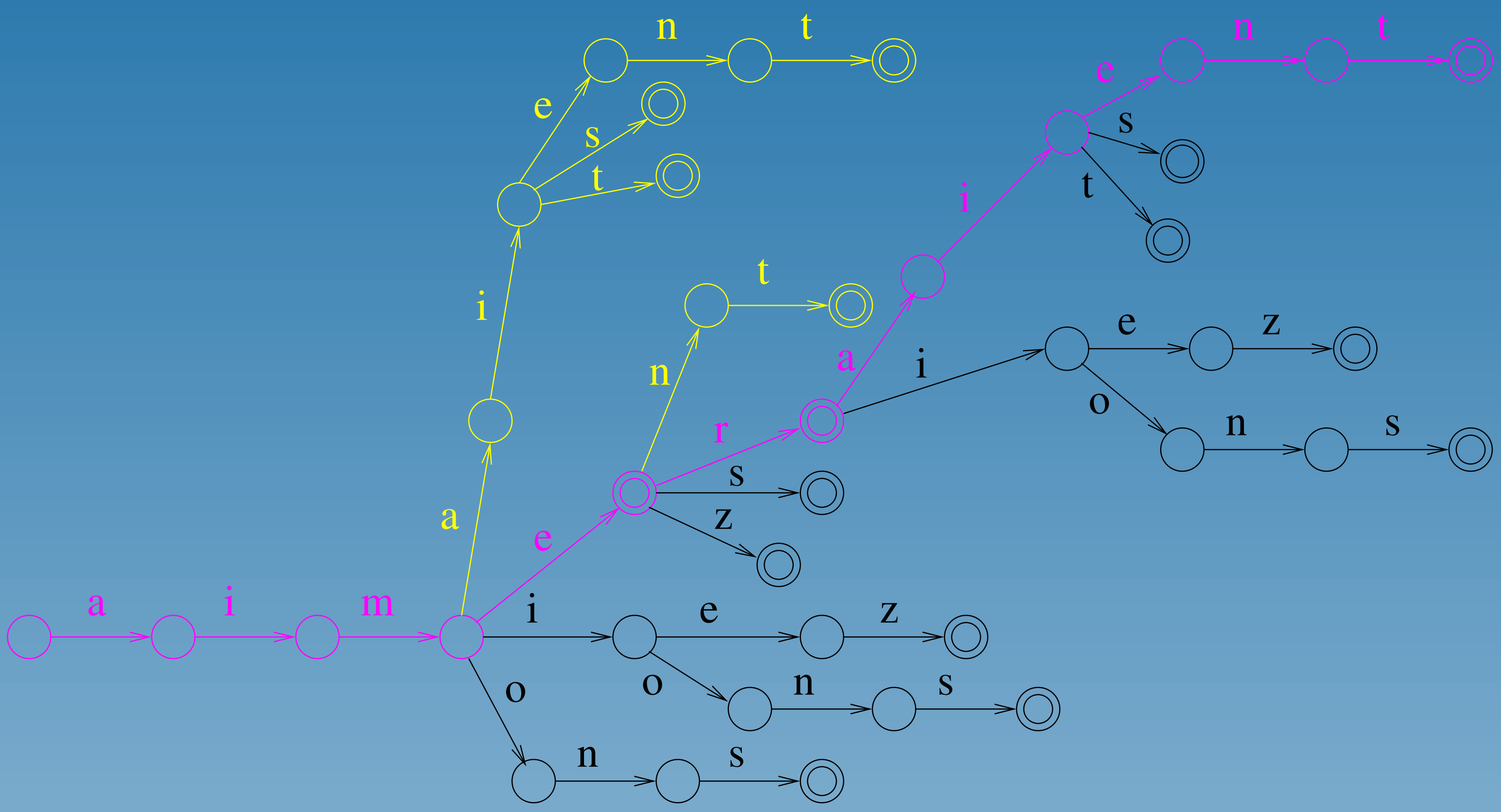

Incremental construction from sorted data

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function sorted_construction;
  $w' \leftarrow \epsilon$;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
    while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
      $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
    end while;
    if $i \leq |w'|$ then repl_or_reg$(\delta(s, w_i), w'_{i+1 \ldots |w'|})$; end if;
    while $i \leq |w|$ do
      $\delta(s, w_i) \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
  end while;
  repl_or_reg$(q_0, w')$;
end function;
function repl_or_reg$(q, v)$;
  if $v \neq \epsilon$ then
    $\delta(q, v_1) \leftarrow$ repl_or_reg$(\delta(q, v_1), v_{2 \ldots |v|})$;
  end if;
  if $\exists_{r \in R}\, r \equiv q$ then
    delete $q$; return $r$;
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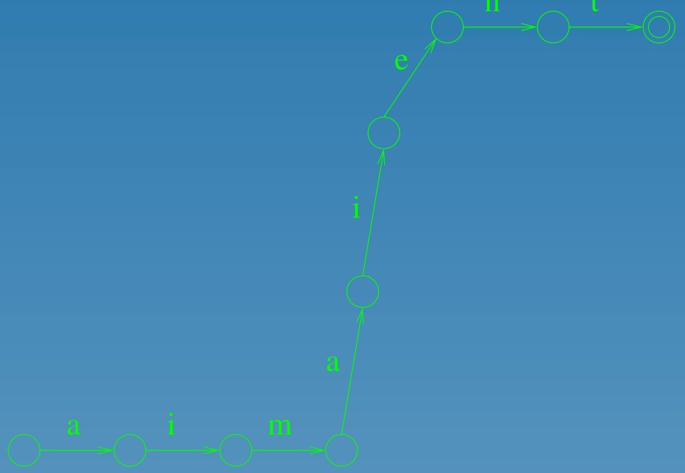

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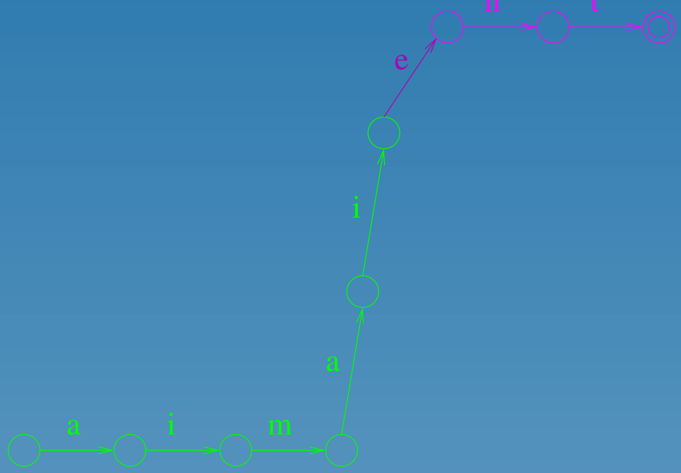

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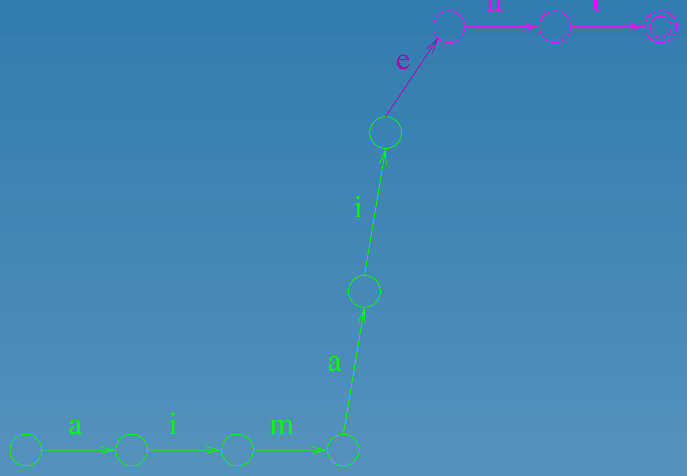
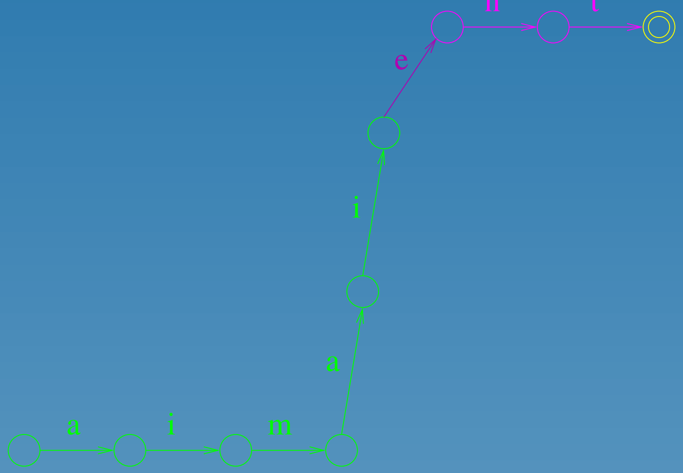
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
    end while;
    repl_or_reg($q_0, w'$);
end function;
function repl_or_reg($q, v$);
    if $v \neq \epsilon$ then
        $\delta(q, v_1) \leftarrow$ repl_or_reg($\delta(q, v_1), v_{2\ldots|v|}$);
    end if;
    if $\exists_{r \in R}\, r \equiv q$ then
        delete $q$; return $r$;
    else
        $R \leftarrow R \cup \{q\}$; return $q$;
    end if;
end function;
```



aimais

Incremental construction from sorted data – examples

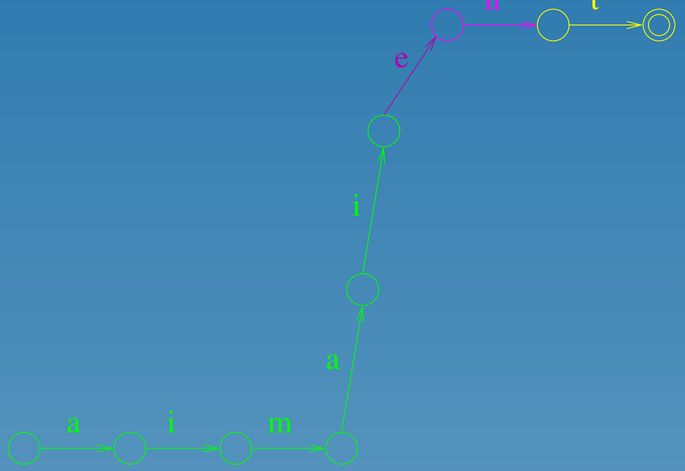
```
function sorted_construction;
    $w' \leftarrow \epsilon$;
    while input not empty do
        $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
        while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        if $i \leq |w'|$ then
            repl_or_reg$(\delta(s, w_i), w'_{i+1\ldots|w'|})$;
        end if;
        while $i \leq |w|$ do
            $\delta(s, w_i) \leftarrow$ new state;
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
    end while;
    repl_or_reg$(q_0, w')$;
end function;
function repl_or_reg$(q, v)$;
    if $v \neq \epsilon$ then
        $\delta(q, v_1) \leftarrow$ repl_or_reg$(\delta(q, v_1), v_{2\ldots|v|})$;
    end if;
    if $\exists_{r \in R}\, r \equiv q$ then
        delete $q$; return $r$;
    else
        $R \leftarrow R \cup \{q\}$; return $q$;
    end if;
end function;
```



aimais

Incremental construction from sorted data – examples

```
function sorted_construction;
   $w' \leftarrow \epsilon$;
   while input not empty do
      $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
      while $i \le |w|$ and $\delta(s, w_i)! = \bot$ do
         $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
      end while;
      if $i \le |w'|$ then
         repl_or_reg($\delta(s, w_i), w'_{i+1...|w'|}$);
      end if;
      while $i \le |w|$ do
         $\delta(s, w_i) \leftarrow$ new state;
         $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
      end while;
      $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
   end while;
   repl_or_reg($q_0, w'$);
end function;
function repl_or_reg($q, v$);
   if $v \neq \epsilon$ then
      $\delta(q, v_1) \leftarrow$ repl_or_reg($\delta(q, v_1), v_{2...|v|}$);
   end if;
   if $\exists_{r \in R}\, r \equiv q$ then
      delete $q$; return $r$;
   else
      $R \leftarrow R \cup \{q\}$; return $q$;
   end if;
end function;
```



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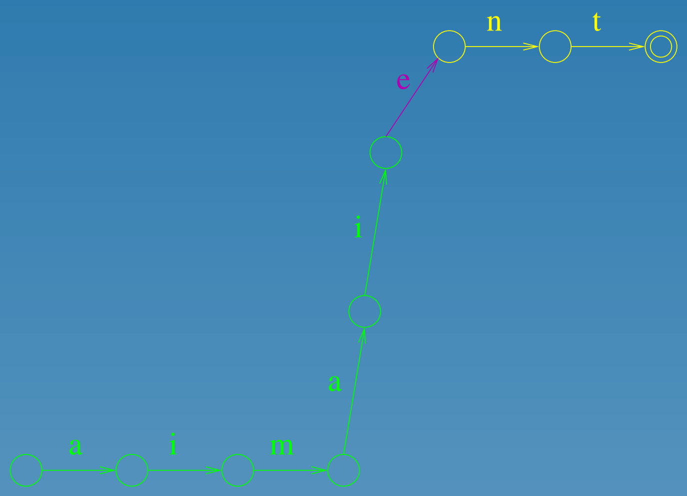
Incremental construction from sorted data – examples

```
function sorted_construction;
   $w' \leftarrow \epsilon$;
   while input not empty do
      $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
      while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
         $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
      end while;
      if $i \leq |w'|$ then
         repl_or_reg$(\delta(s, w_i), w'_{i+1 \ldots |w'|})$;
      end if;
      while $i \leq |w|$ do
         $\delta(s, w_i) \leftarrow$ new state;
         $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
      end while;
      $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
   end while;
   repl_or_reg$(q_0, w')$;
end function;
function repl_or_reg$(q, v)$;
   if $v \neq \epsilon$ then
      $\delta(q, v_1) \leftarrow$ repl_or_reg$(\delta(q, v_1), v_{2 \ldots |v|})$;
   end if;
   if $\exists_{r \in R}\, r \equiv q$ then
      delete $q$; return $r$;
   else
      $R \leftarrow R \cup \{q\}$; return $q$;
   end if;
end function;
```



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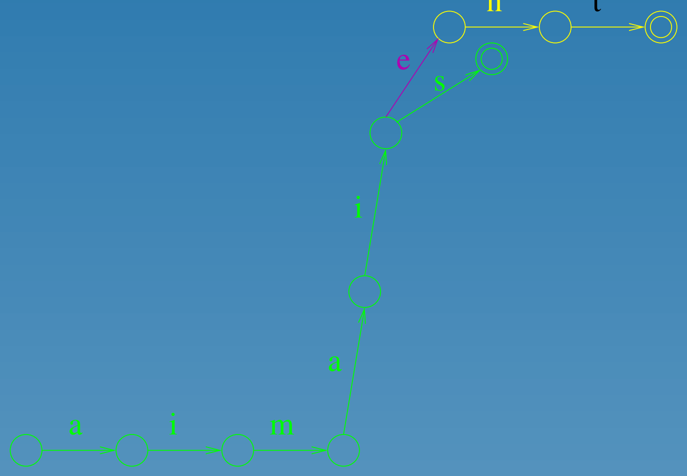
Incremental construction from sorted data – examples

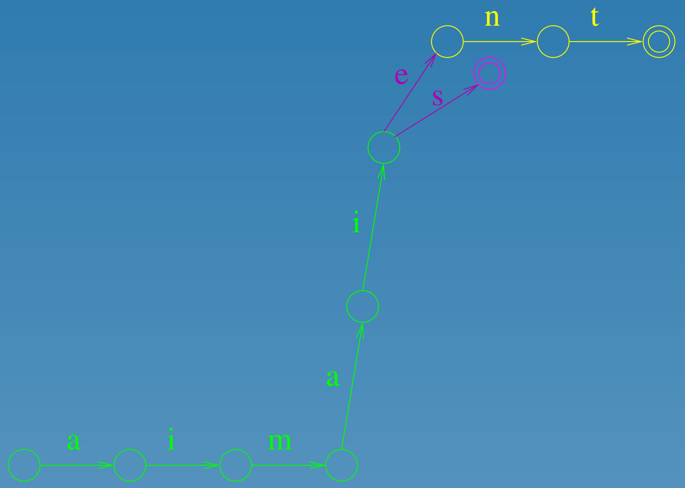
```
function sorted_construction;
    $w' \leftarrow \epsilon$;
    while input not empty do
        $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
        while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        if $i \leq |w'|$ then
            repl_or_reg$(\delta(s, w_i), w'_{i+1\ldots|w'|})$;
        end if;
        while $i \leq |w|$ do
            $\delta(s, w_i) \leftarrow$ new state;
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
    end while;
    repl_or_reg$(q_0, w')$;
end function;
function repl_or_reg$(q, v)$;
    if $v \neq \epsilon$ then
        $\delta(q, v_1) \leftarrow$ repl_or_reg$(\delta(q, v_1), v_{2\ldots|v|})$;
    end if;
    if $\exists_{r \in R}\, r \equiv q$ then
        delete $q$; return $r$;
    else
        $R \leftarrow R \cup \{q\}$; return $q$;
    end if;
end function;
```



aimais

Incremental construction from sorted data – examples

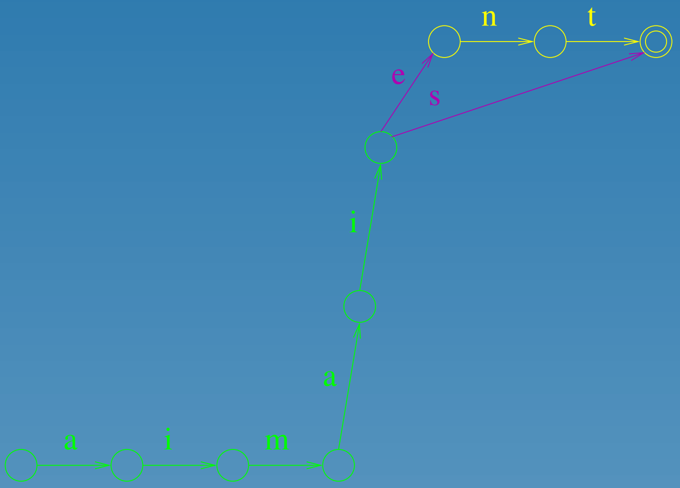
```
function sorted_construction;
   $w' \leftarrow \epsilon$;
   while input not empty do
      $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
      while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
         $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
      end while;
      if $i \leq |w'|$ then
         repl_or_reg($\delta(s, w_i), w'_{i+1\ldots|w'|}$);
      end if;
      while $i \leq |w|$ do
         $\delta(s, w_i) \leftarrow$ new state;
         $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
      end while;
      $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
   end while;
   repl_or_reg($q_0, w'$);
end function;
function repl_or_reg($q, v$);
   if $v \neq \epsilon$ then
      $\delta(q, v_1) \leftarrow$ repl_or_reg($\delta(q, v_1), v_{2\ldots|v|}$);
   end if;
   if $\exists_{r \in R}\, r \equiv q$ then
      delete $q$; return $r$;
   else
      $R \leftarrow R \cup \{q\}$; return $q$;
   end if;
end function;
```



aimait

Incremental construction from sorted data – examples

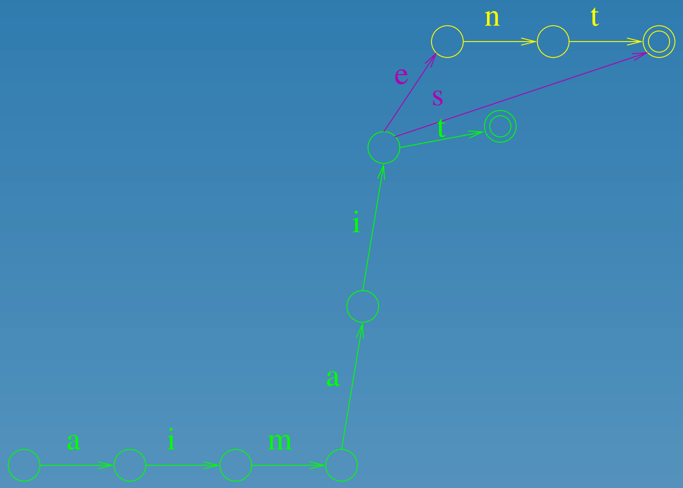
```
function sorted_construction;
    $w' \leftarrow \epsilon$;
    while input not empty do
        $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
        while $i \leq |w|$ and $\delta(s, w_i) != \perp$ do
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        if $i \leq |w'|$ then
            repl_or_reg$(\delta(s, w_i), w'_{i+1\ldots|w'|})$;
        end if;
        while $i \leq |w|$ do
            $\delta(s, w_i) \leftarrow$ new state;
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
    end while;
    repl_or_reg$(q_0, w')$;
end function;
function repl_or_reg$(q, v)$;
    if $v \neq \epsilon$ then
        $\delta(q, v_1) \leftarrow$ repl_or_reg$(\delta(q, v_1), v_{2\ldots|v|})$;
    end if;
    if $\exists_{r \in R}\, r \equiv q$ then
        delete $q$; return $r$;
    else
        $R \leftarrow R \cup \{q\}$; return $q$;
    end if;
end function;
```



aimait

Incremental construction from sorted data – examples

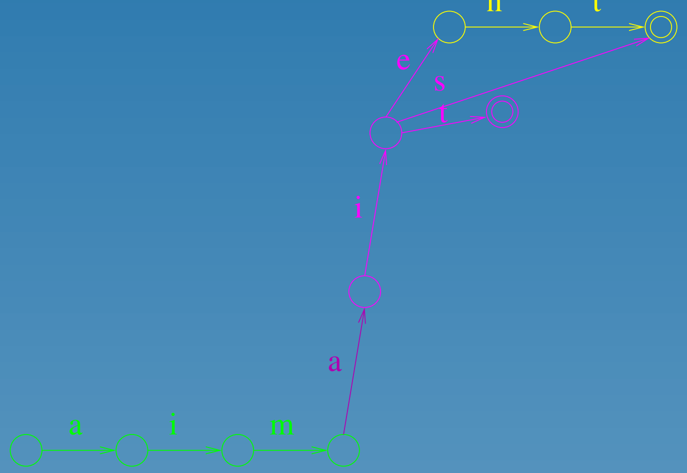
```
function sorted_construction;
   $w' \leftarrow \epsilon$;
   while input not empty do
      $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
      while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
         $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
      end while;
      if $i \leq |w'|$ then
         repl_or_reg$(\delta(s, w_i), w'_{i+1\ldots|w'|})$;
      end if;
      while $i \leq |w|$ do
         $\delta(s, w_i) \leftarrow$ new state;
         $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
      end while;
      $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
   end while;
   repl_or_reg$(q_0, w')$;
end function;
function repl_or_reg$(q, v)$;
   if $v \neq \epsilon$ then
      $\delta(q, v_1) \leftarrow$ repl_or_reg$(\delta(q, v_1), v_{2\ldots|v|})$;
   end if;
   if $\exists_{r \in R}\, r \equiv q$ then
      delete $q$; return $r$;
   else
      $R \leftarrow R \cup \{q\}$; return $q$;
   end if;
end function;
```



aimait

Incremental construction from sorted data – examples

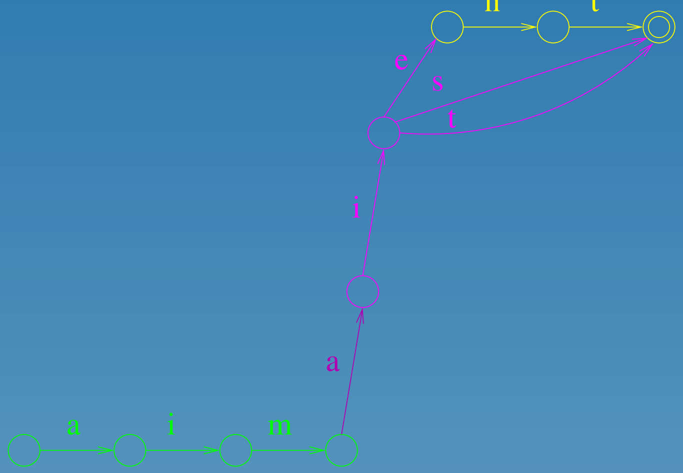
```
function sorted_construction;
  $w' \leftarrow \epsilon$;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
    while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
      $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
    end while;
    if $i \leq |w'|$ then
      repl_or_reg$(\delta(s, w_i), w'_{i+1\ldots|w'|})$;
    end if;
    while $i \leq |w|$ do
      $\delta(s, w_i) \leftarrow$ new state;
      $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
  end while;
  repl_or_reg$(q_0, w')$;
end function;
function repl_or_reg$(q, v)$;
  if $v \neq \epsilon$ then
    $\delta(q, v_1) \leftarrow$ repl_or_reg$(\delta(q, v_1), v_{2\ldots|v|})$;
  end if;
  if $\exists_{r \in R}\, r \equiv q$ then
    delete $q$; return $r$;
  else
    $R \leftarrow R \cup \{q\}$; return $q$;
  end if;
end function;
```



aime

Incremental construction from sorted data – examples

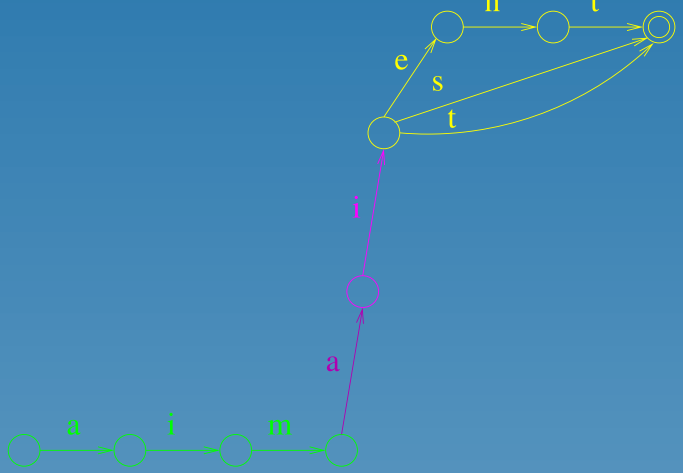
```
function sorted_construction;
    $w' \leftarrow \epsilon$;
    while input not empty do
        $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
        while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        if $i \leq |w'|$ then
            repl_or_reg$(\delta(s, w_i), w'_{i+1\ldots|w'|})$;
        end if;
        while $i \leq |w|$ do
            $\delta(s, w_i) \leftarrow$ new state;
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
    end while;
    repl_or_reg$(q_0, w')$;
end function;
function repl_or_reg$(q, v)$;
    if $v \neq \epsilon$ then
        $\delta(q, v_1) \leftarrow$ repl_or_reg$(\delta(q, v_1), v_{2\ldots|v|})$;
    end if;
    if $\exists_{r \in R}\, r \equiv q$ then
        delete $q$; return $r$;
    else
        $R \leftarrow R \cup \{q\}$; return $q$;
    end if;
end function;
```



aime

Incremental construction from sorted data – examples

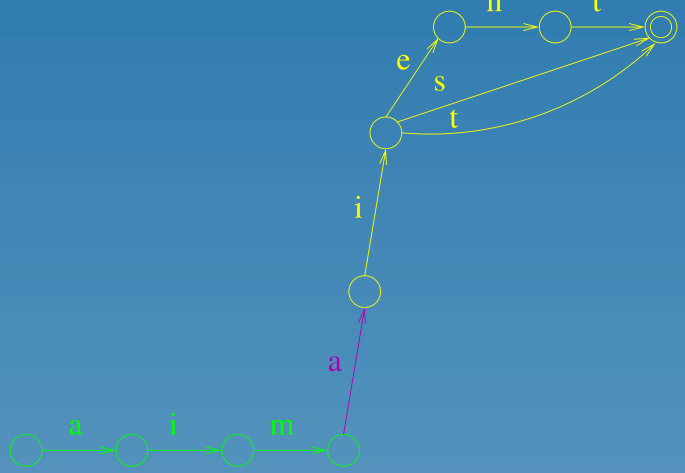
```
function sorted_construction;
    $w' \leftarrow \epsilon$;
    while input not empty do
        $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
        while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        if $i \leq |w'|$ then
            repl_or_reg($\delta(s, w_i), w'_{i+1\ldots|w'|}$);
        end if;
        while $i \leq |w|$ do
            $\delta(s, w_i) \leftarrow$ new state;
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
    end while;
    repl_or_reg($q_0, w'$);
end function;
function repl_or_reg($q, v$);
    if $v \neq \epsilon$ then
        $\delta(q, v_1) \leftarrow$ repl_or_reg($\delta(q, v_1), v_{2\ldots|v|}$);
    end if;
    if $\exists_{r \in R}\, r \equiv q$ then
        delete $q$; return $r$;
    else
        $R \leftarrow R \cup \{q\}$; return $q$;
    end if;
end function;
```



aime

Incremental construction from sorted data – examples

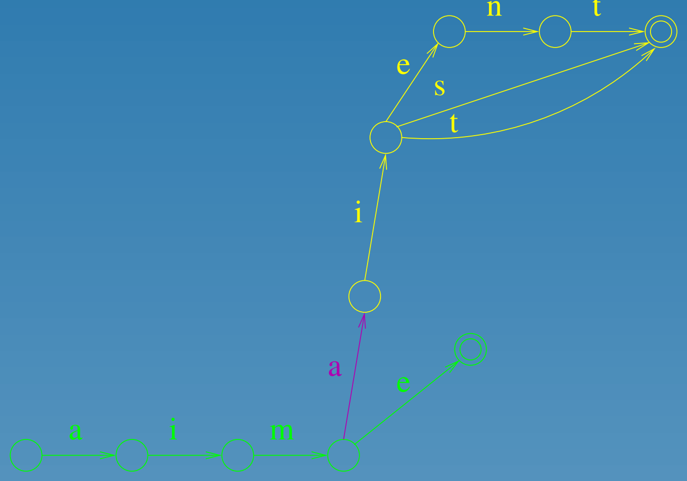
```
function sorted_construction;
   $w' \leftarrow \epsilon$;
   while input not empty do
      $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
      while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
         $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
      end while;
      if $i \leq |w'|$ then
         repl_or_reg$(\delta(s, w_i), w'_{i+1\ldots|w'|})$;
      end if;
      while $i \leq |w|$ do
         $\delta(s, w_i) \leftarrow$ new state;
         $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
      end while;
      $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
   end while;
   repl_or_reg$(q_0, w')$;
end function;
function repl_or_reg$(q, v)$;
   if $v \neq \epsilon$ then
      $\delta(q, v_1) \leftarrow$ repl_or_reg$(\delta(q, v_1), v_{2\ldots|v|})$;
   end if;
   if $\exists_{r \in R}\, r \equiv q$ then
      delete $q$; return $r$;
   else
      $R \leftarrow R \cup \{q\}$; return $q$;
   end if;
end function;
```



aime

Incremental construction from sorted data – examples

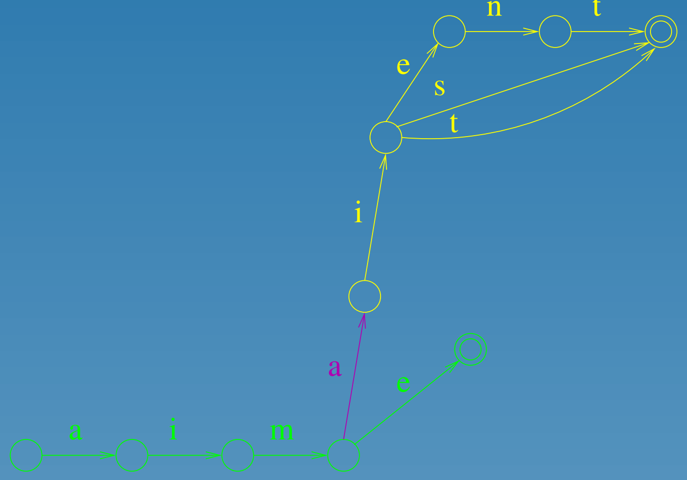
```
function sorted_construction;
    $w' \leftarrow \epsilon$;
    while input not empty do
        $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
        while $i \le |w|$ and $\delta(s, w_i)! = \perp$ do
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        if $i \le |w'|$ then
            repl_or_reg($\delta(s, w_i), w'_{i+1\ldots|w'|}$);
        end if;
        while $i \le |w|$ do
            $\delta(s, w_i) \leftarrow$ new state;
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
    end while;
    repl_or_reg($q_0, w'$);
end function;
function repl_or_reg($q, v$);
    if $v \neq \epsilon$ then
        $\delta(q, v_1) \leftarrow$ repl_or_reg($\delta(q, v_1), v_{2\ldots|v|}$);
    end if;
    if $\exists_{r \in R}\, r \equiv q$ then
        delete $q$; return $r$;
    else
        $R \leftarrow R \cup \{q\}$; return $q$;
    end if;
end function;
```



aime

Incremental construction from sorted data – examples

```
function sorted_construction;
  $w' \leftarrow \epsilon$;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
    while $i \le |w|$ and $\delta(s, w_i)! = \perp$ do
      $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
    end while;
    if $i \le |w'|$ then
      repl_or_reg($\delta(s, w_i), w'_{i+1\ldots|w'|}$);
    end if;
    while $i \le |w|$ do
      $\delta(s, w_i) \leftarrow$ new state;
      $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
  end while;
  repl_or_reg($q_0, w'$);
end function;
function repl_or_reg($q, v$);
  if $v \neq \epsilon$ then
    $\delta(q, v_1) \leftarrow$ repl_or_reg($\delta(q, v_1), v_{2\ldots|v|}$);
  end if;
  if $\exists_{r \in R}\, r \equiv q$ then
    delete $q$; return $r$;
  else
    $R \leftarrow R \cup \{q\}$; return $q$;
  end if;
end function;
```



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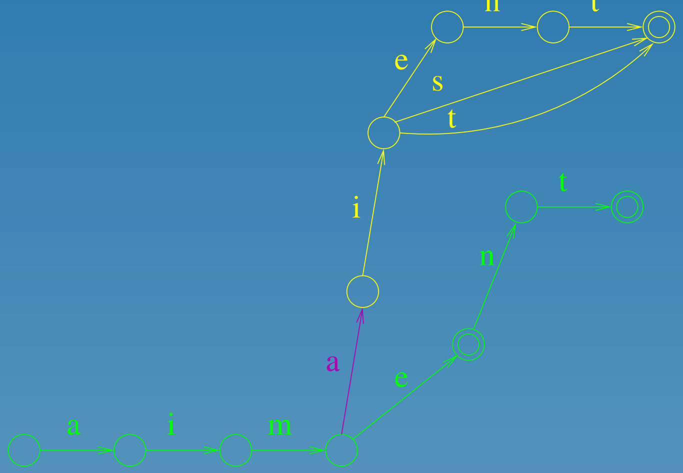
Incremental construction from sorted data – examples

```
function sorted_construction;
    $w' \leftarrow \epsilon$;
    while input not empty do
        $s \leftarrow q_0$; $i \leftarrow 1$; $w \leftarrow$ next word;
        while $i \leq |w|$ and $\delta(s, w_i)! = \perp$ do
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        if $i \leq |w'|$ then
            repl_or_reg($\delta(s, w_i), w'_{i+1 \ldots |w'|}$);
        end if;
        while $i \leq |w|$ do
            $\delta(s, w_i) \leftarrow$ new state;
            $s \leftarrow \delta(s, w_i)$; $i \leftarrow i + 1$;
        end while;
        $F \leftarrow F \cup \{s\}$; $w' \leftarrow w$
    end while;
    repl_or_reg($q_0, w'$);
end function;
function repl_or_reg($q, v$);
    if $v \neq \epsilon$ then
        $\delta(q, v_1) \leftarrow$ repl_or_reg($\delta(q, v_1), v_{2 \ldots |v|}$);
    end if;
    if $\exists_{r \in R}\, r \equiv q$ then
        delete $q$; return $r$;
    else
        $R \leftarrow R \cup \{q\}$; return $q$;
    end if;
end function;
```



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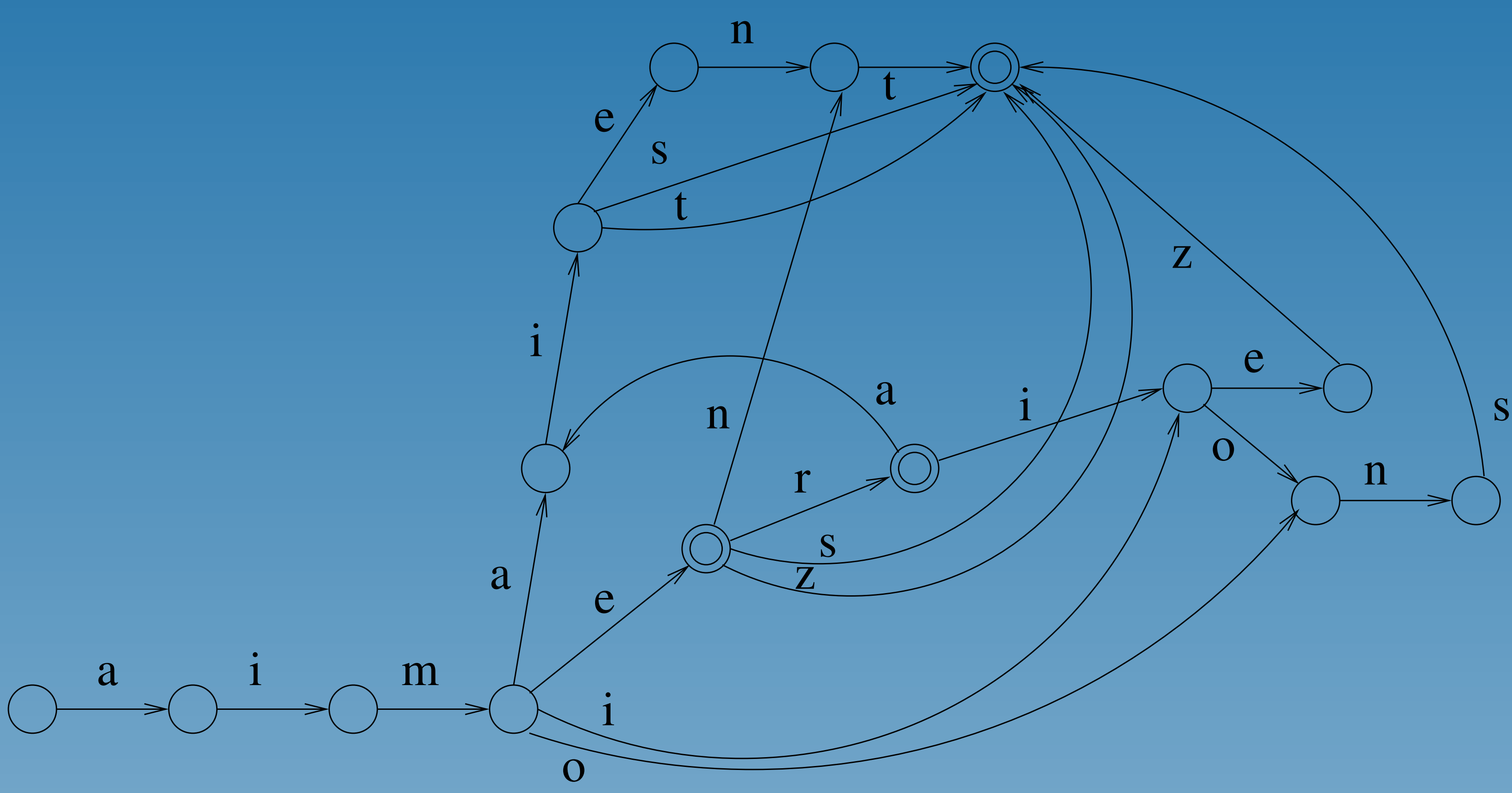
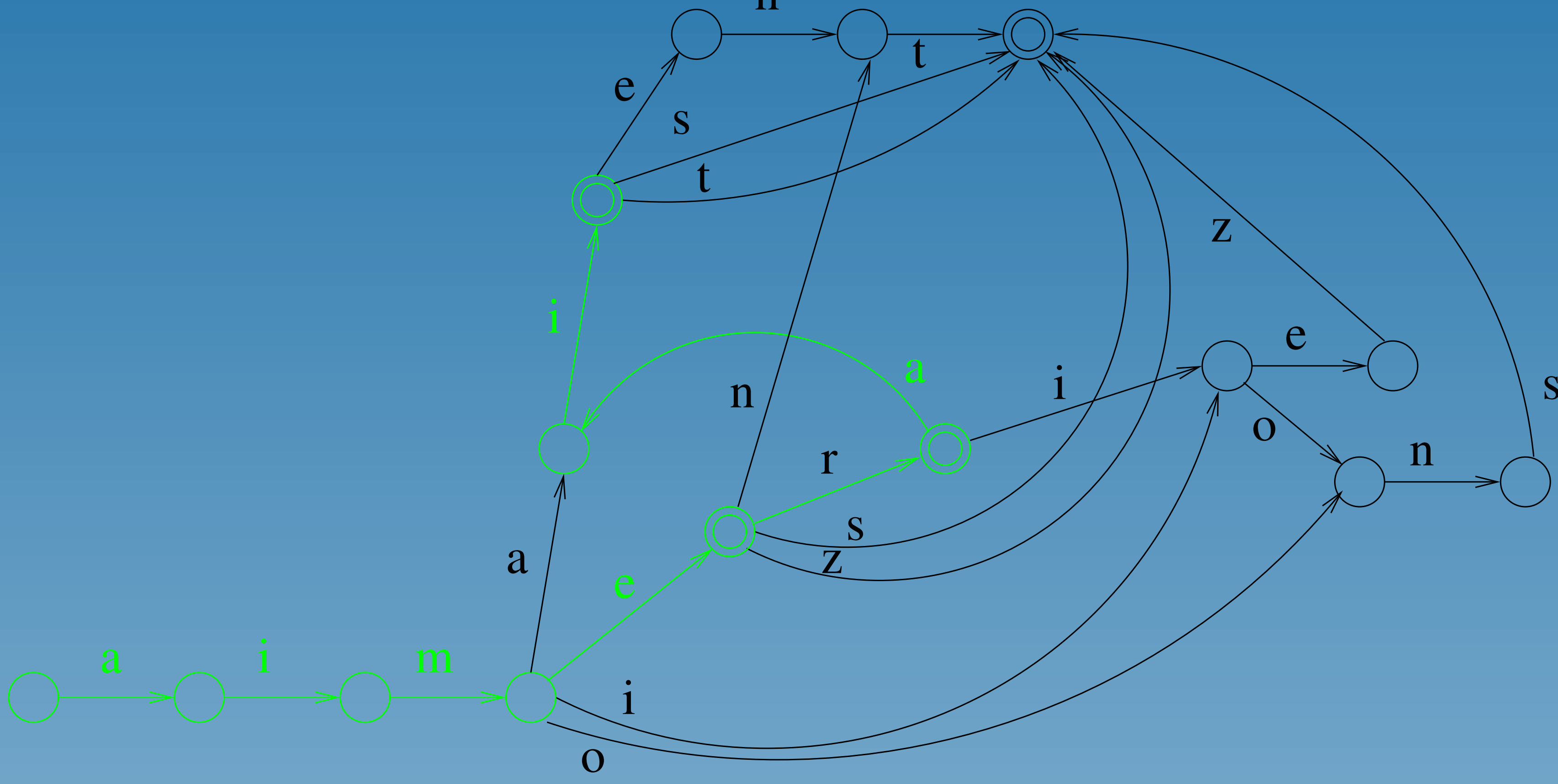
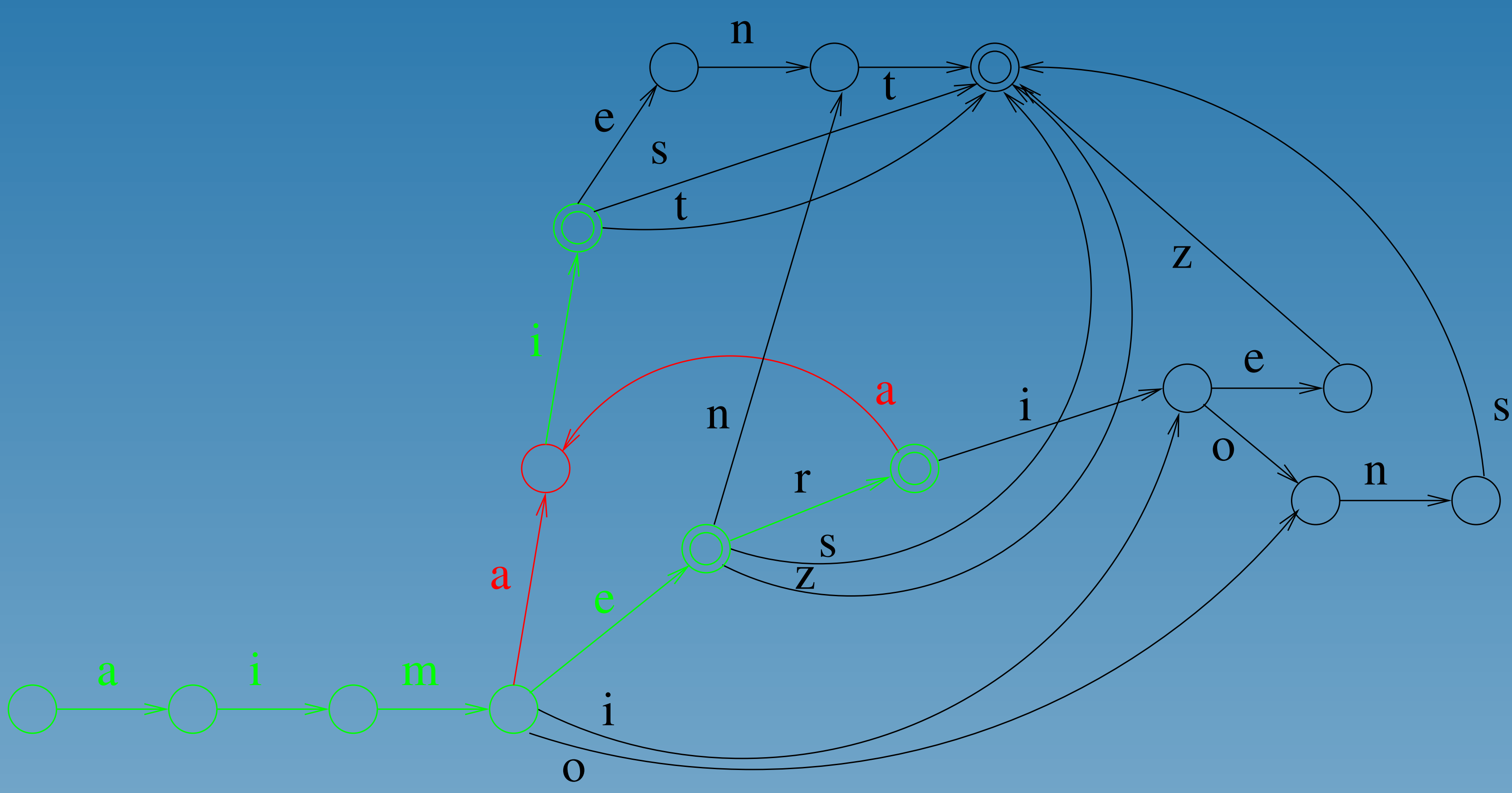
Unsorted data – hidden dangers



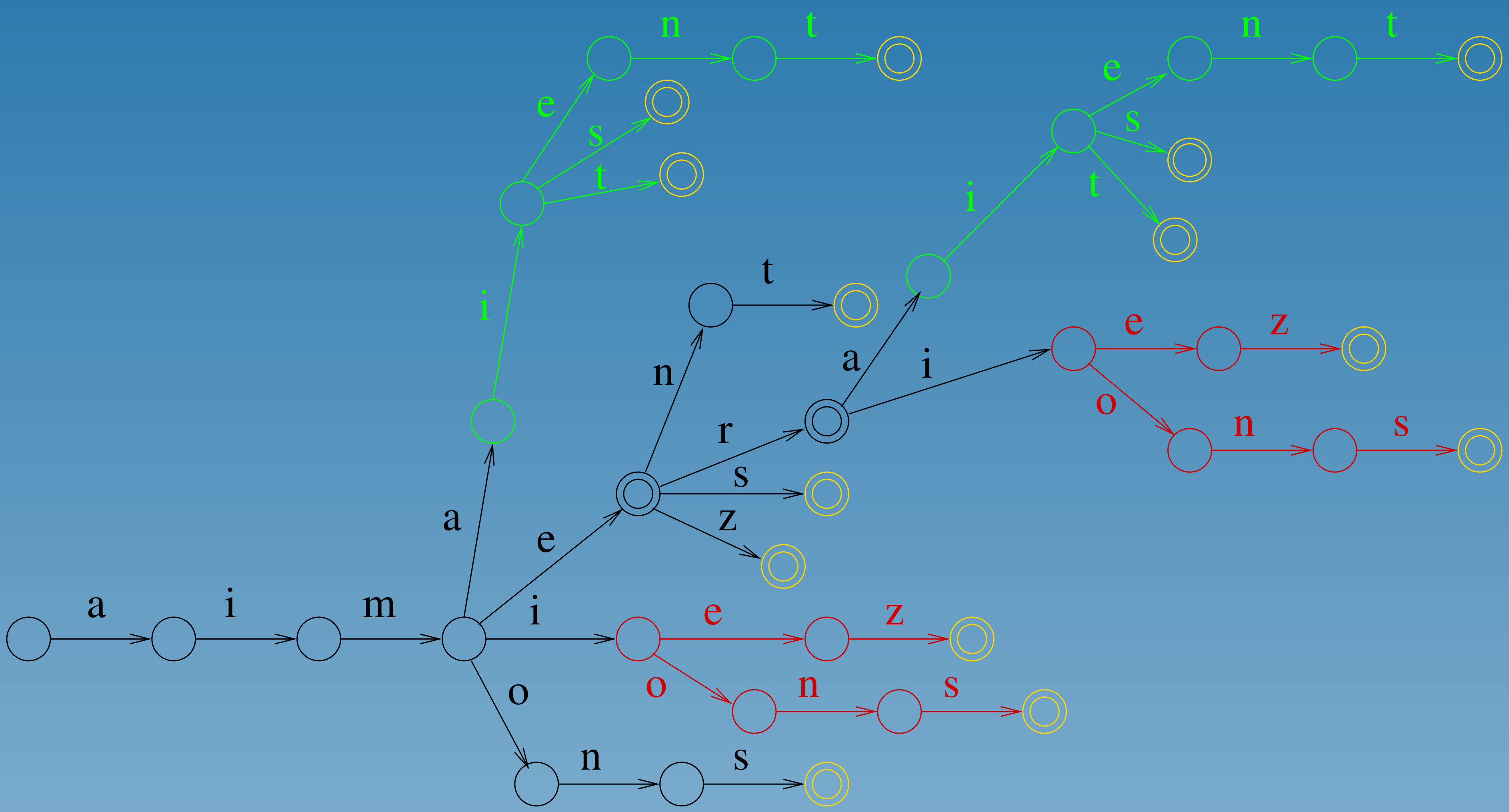
Unsorted data – hidden dangers



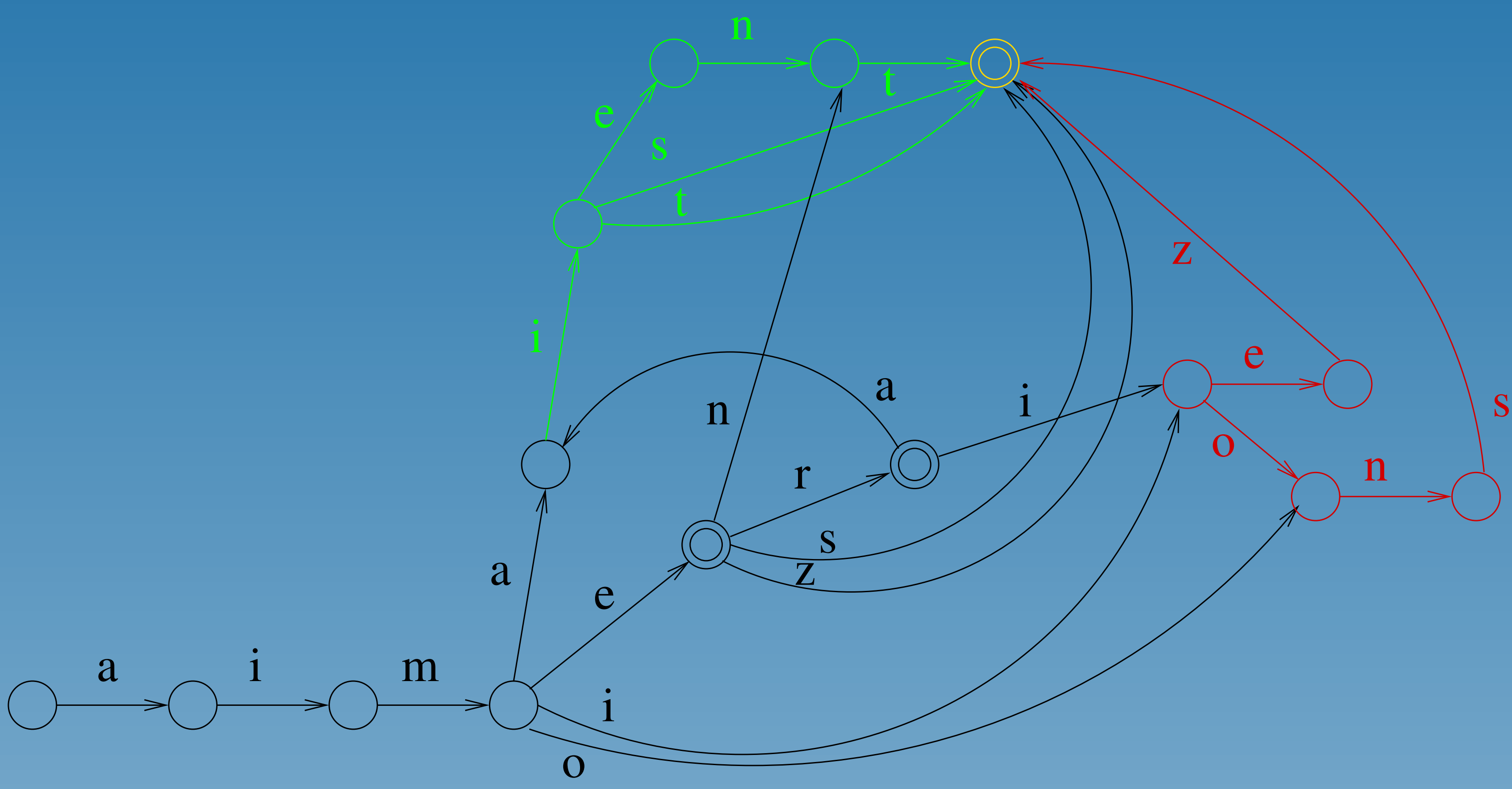
Unsorted data – hidden dangers



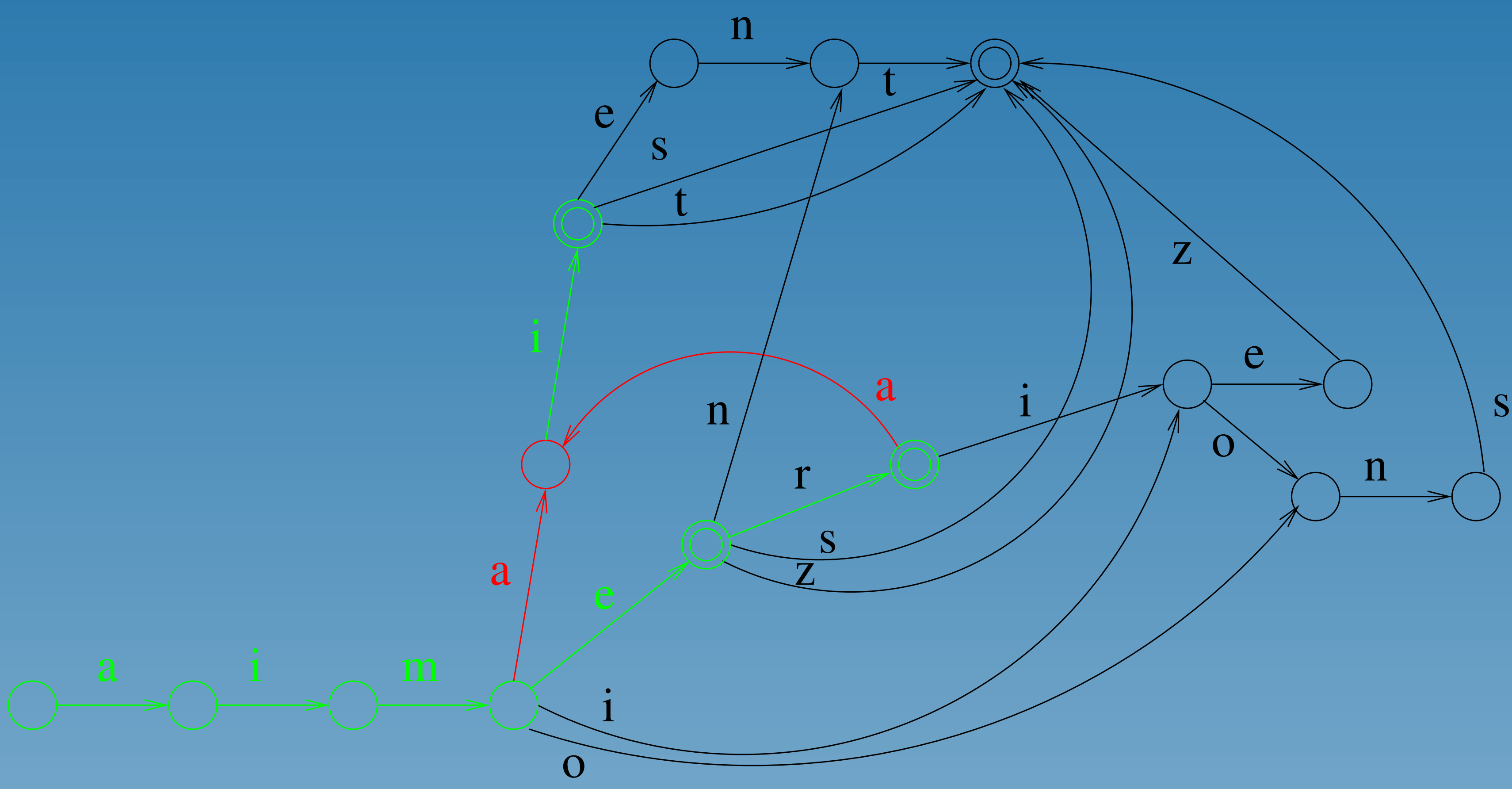
Unsorted data – hidden dangers



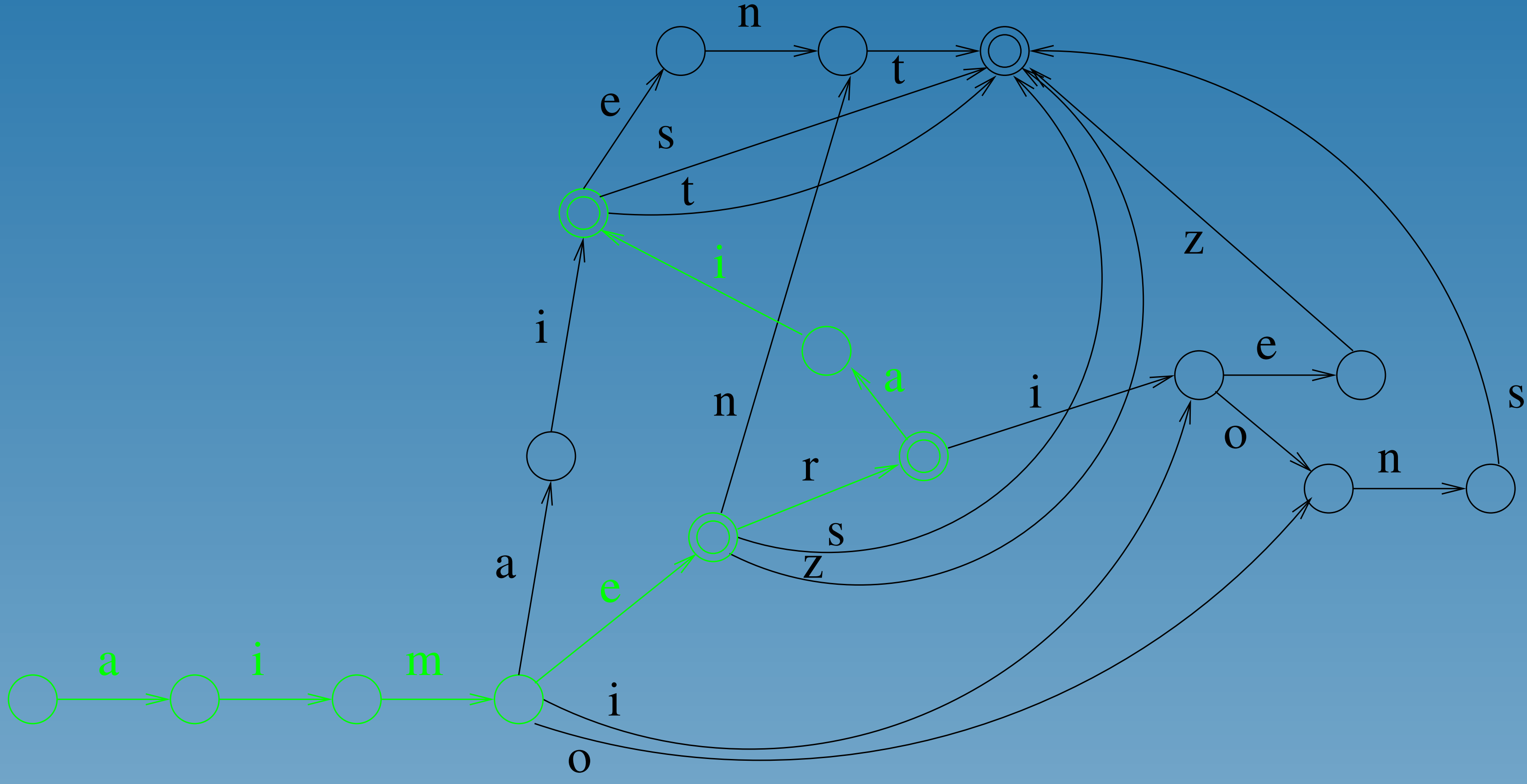
Unsorted data – hidden dangers



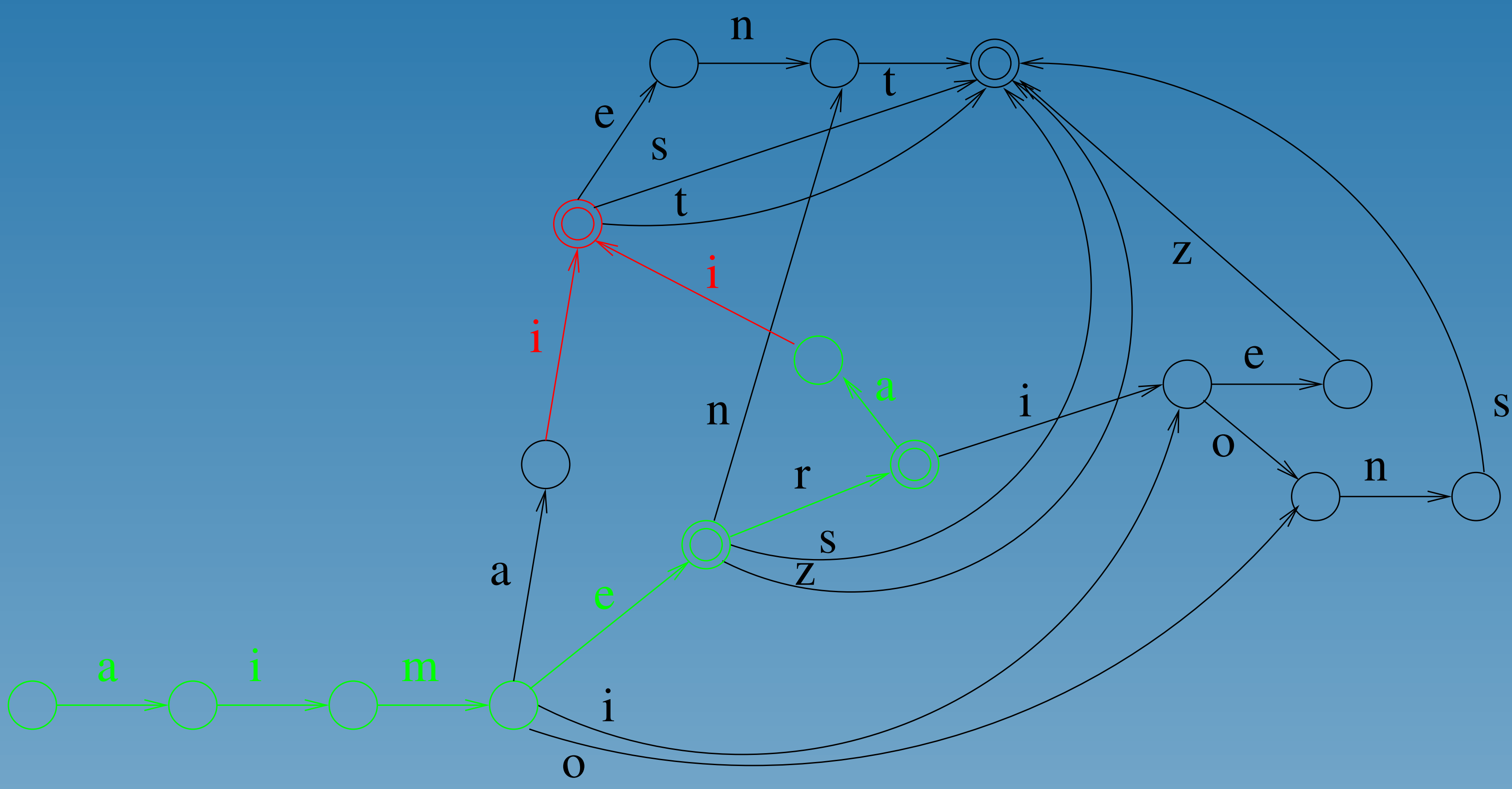
Unsorted data – hidden dangers



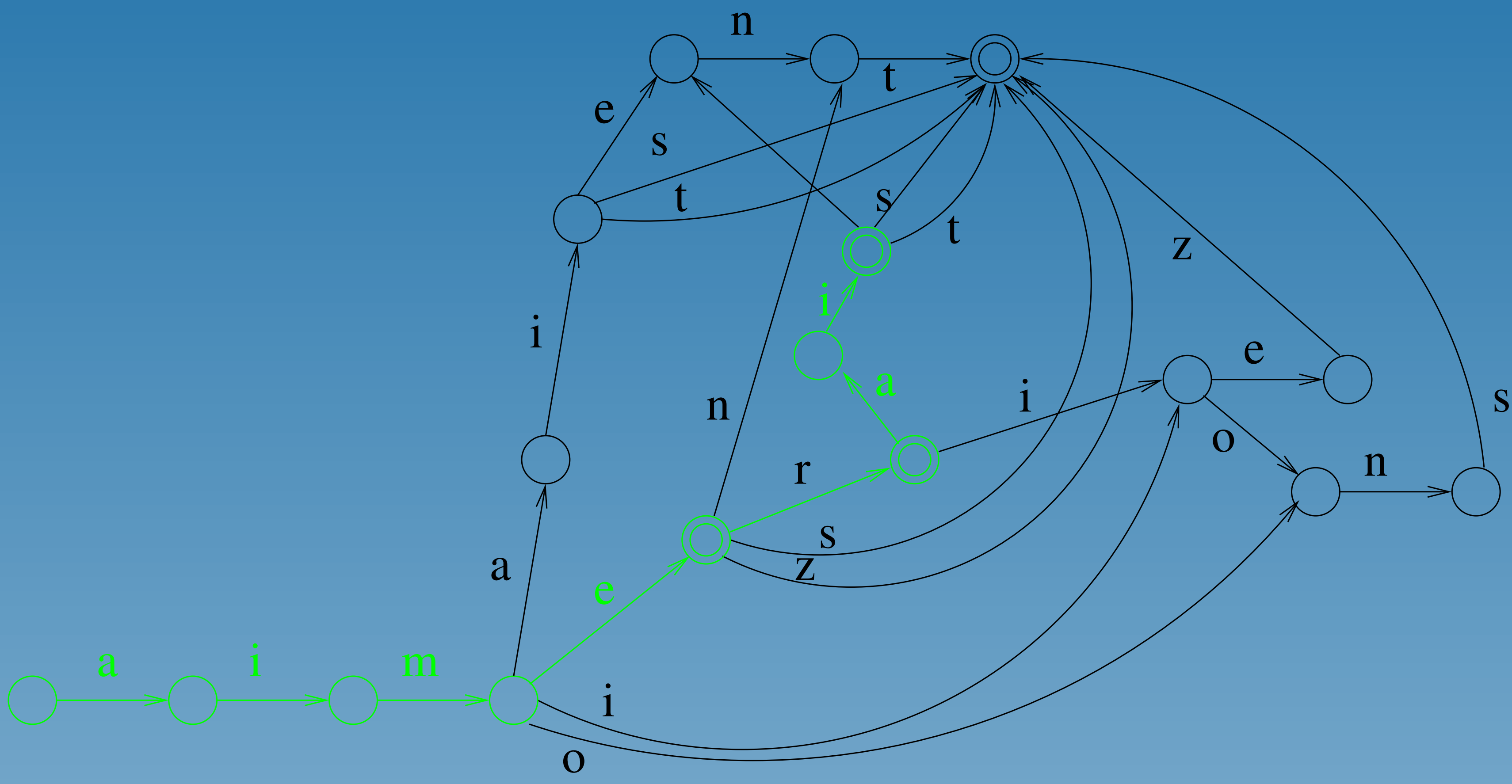
Unsorted data – hidden dangers



Unsorted data – hidden dangers



Unsorted data – hidden dangers



Incremental construction from unsorted data

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$; end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$; if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```

Incremental construction from unsorted data

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
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    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R} r \equiv s$ then
        if $i = u$ and $i > 0$ then $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$; end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$; if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```

Incremental construction from unsorted data

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$; end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$; if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```

Incremental construction from unsorted data

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$; end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$; if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```

Incremental construction from unsorted data

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R} r \equiv s$ then
        if $i = u$ and $i > 0$ then $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$; end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$; if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```

Incremental construction from unsorted data

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$; end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$; if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```

Incremental construction from unsorted data

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then $R \leftarrow R \setminus \{\text{top}(P)\}$; $u \leftarrow u - 1$; end if;
        delete $s$; $\delta(\text{top}(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$; if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```

Incremental construction from unsorted data

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R} r \equiv s$ then
        if $i = u$ and $i > 0$ then $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$; end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$; if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```

Incr. constr. from unsorted data – examples

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
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      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
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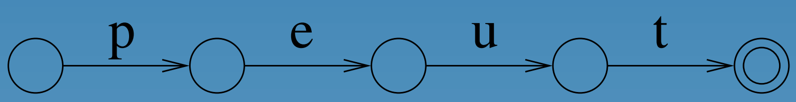
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



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Incr. constr. from unsorted data – examples

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
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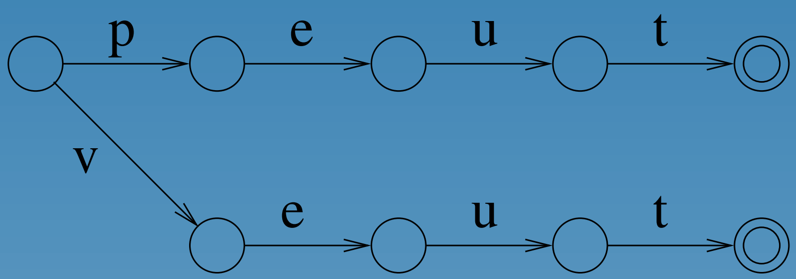
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



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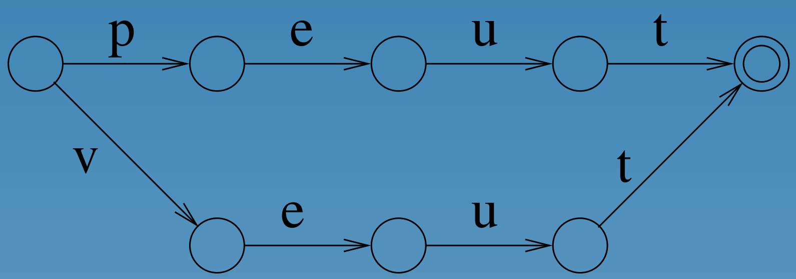
Incr. constr. from unsorted data – examples

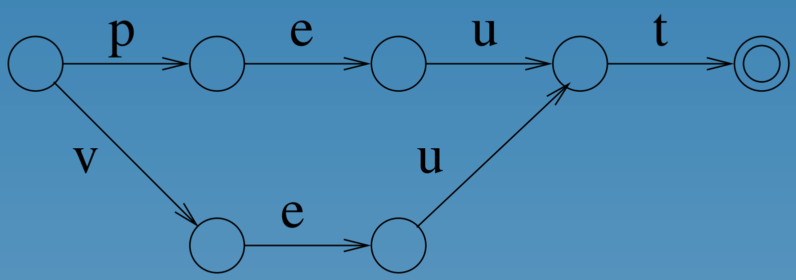
```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



veut

Incr. constr. from unsorted data – examples

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R} r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



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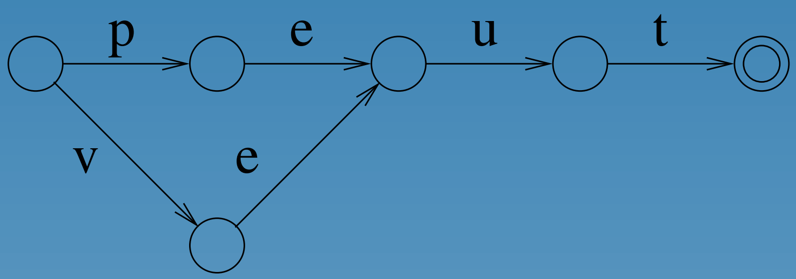
Incr. constr. from unsorted data – examples

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R} r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



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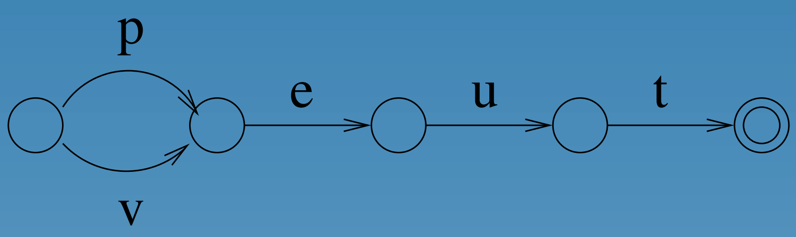
Incr. constr. from unsorted data – examples

```
function unsorted_construction;
   while input not empty do
      $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
      while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
      and fanin$(\delta(s, w_i)) \leq 1$ do
         $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
      end while;
      $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
      while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
         $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
         push$(s, P)$; $i \leftarrow i + 1$;
      end while;
      while $i \leq |w|$ do
         $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
         push$(s, P)$; $i \leftarrow i + 1$;
      end while;
      $F \leftarrow F \cup \{s\}$; pop$(P)$;
      while $P$ not empty do
         if $\exists_{r \in R} r \equiv s$ then
            if $i = u$ and $i > 0$ then
               $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
            end if;
            delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
         else
            $R \leftarrow R \cup \{s\}$;
            if $i = u$ then break; end if;
         end if;
         $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
      end while;
      reset $P$;
   end while;
end function;
```



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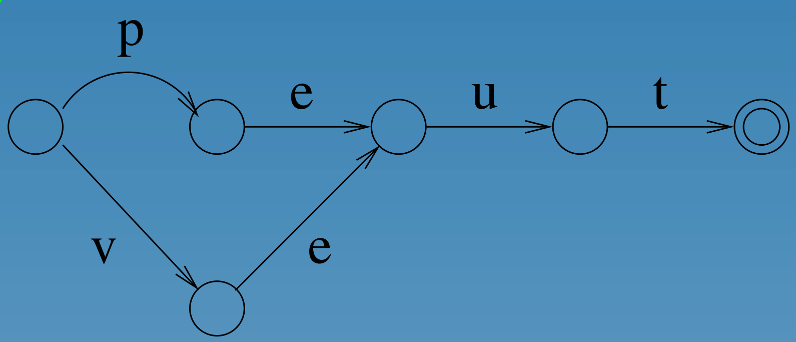
Incr. constr. from unsorted data – examples

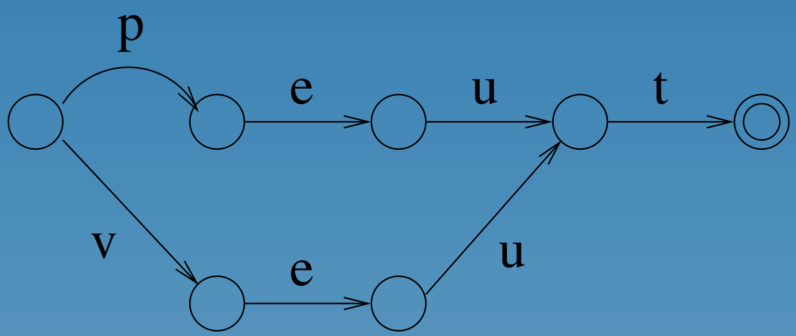
```
function unsorted_construction;
    while input not empty do
        $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
        while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
        and fanin$(\delta(s, w_i)) \leq 1$ do
            $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
        end while;
        $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
        while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
            $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
            push$(s, P)$; $i \leftarrow i + 1$;
        end while;
        while $i \leq |w|$ do
            $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
            push$(s, P)$; $i \leftarrow i + 1$;
        end while;
        $F \leftarrow F \cup \{s\}$; pop$(P)$;
        while $P$ not empty do
            if $\exists_{r \in R}\, r \equiv s$ then
                if $i = u$ and $i > 0$ then
                    $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
                end if;
                delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
            else
                $R \leftarrow R \cup \{s\}$;
                if $i = u$ then break; end if;
            end if;
            $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
        end while;
        reset $P$;
    end while;
end function;
```



veux

Incr. constr. from unsorted data – examples

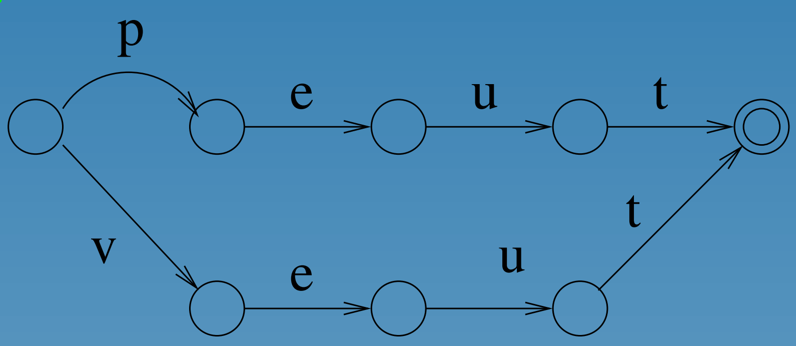
```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



veux

Incr. constr. from unsorted data – examples

```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



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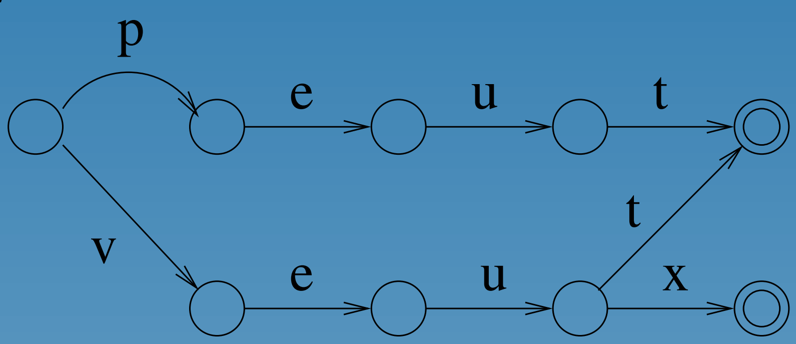
Incr. constr. from unsorted data – examples

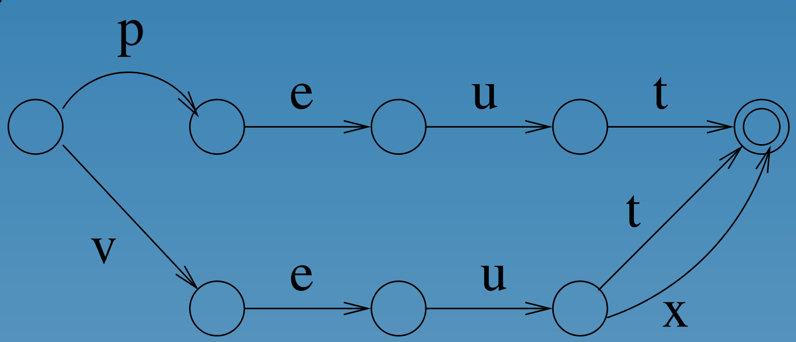
```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



veux

Incr. constr. from unsorted data – examples

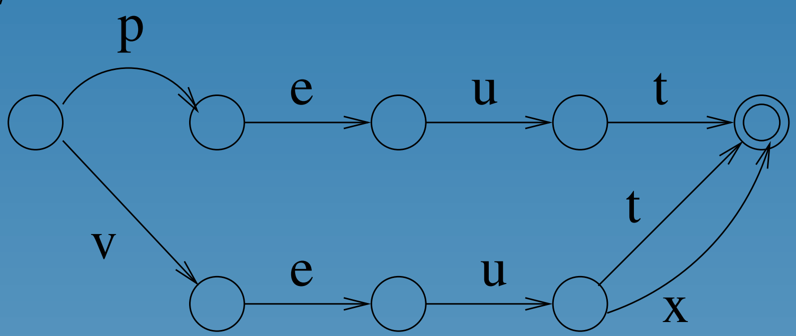
```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



veux

Incr. constr. from unsorted data – examples

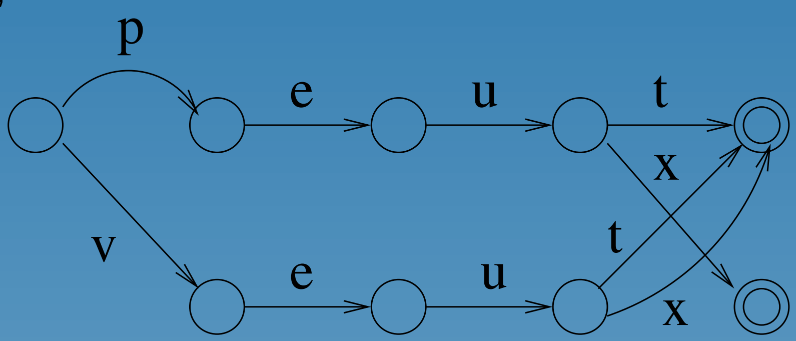
```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



peux

Incr. constr. from unsorted data – examples

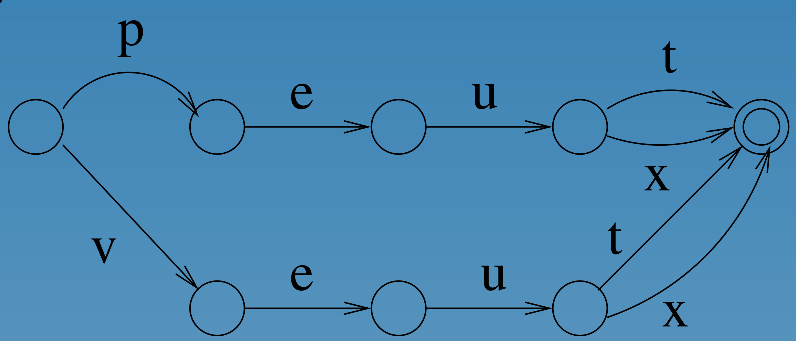
```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



peux

Incr. constr. from unsorted data – examples

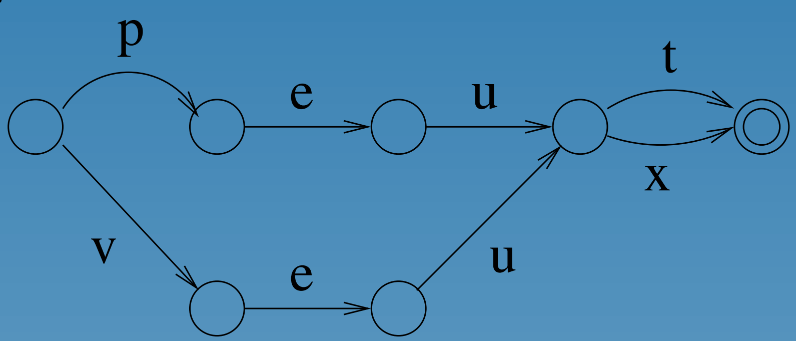
```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



veux

Incr. constr. from unsorted data – examples

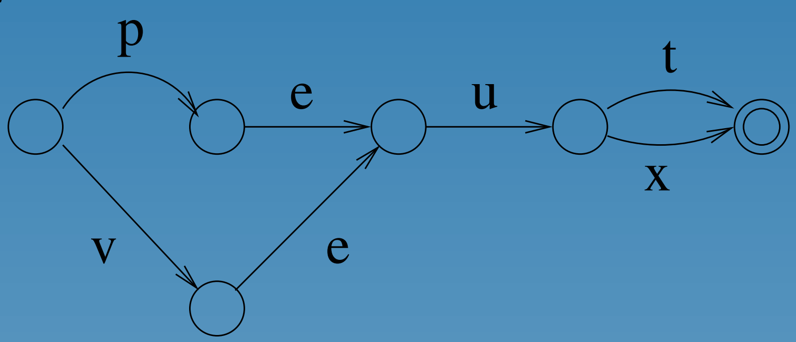
```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \perp$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



veux

Incr. constr. from unsorted data – examples

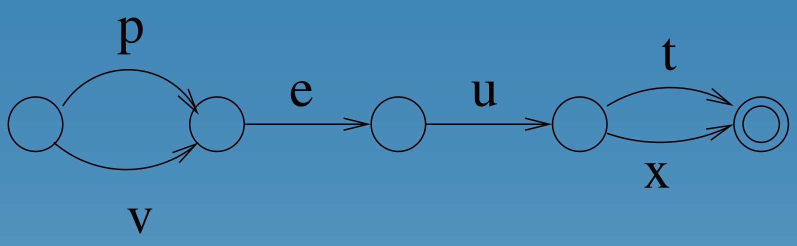
```
function unsorted_construction;
  while input not empty do
    $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$
    and fanin$(\delta(s, w_i)) \leq 1$ do
      $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
    while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ do
      $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    while $i \leq |w|$ do
      $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
      push$(s, P)$; $i \leftarrow i + 1$;
    end while;
    $F \leftarrow F \cup \{s\}$; pop$(P)$;
    while $P$ not empty do
      if $\exists_{r \in R}\, r \equiv s$ then
        if $i = u$ and $i > 0$ then
          $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
        end if;
        delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
      else
        $R \leftarrow R \cup \{s\}$;
        if $i = u$ then break; end if;
      end if;
      $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
    end while;
    reset $P$;
  end while;
end function;
```



veux

Incr. constr. from unsorted data – examples

```
function unsorted_construction;
   while input not empty do
      $s \leftarrow q_0$; $i \leftarrow 0$; $w \leftarrow$ next word; push$(s, P)$;
      while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$
      and fanin$(\delta(s, w_i)) \leq 1$ do
         $s \leftarrow \delta(s, w_i)$; push$(s, P)$; $i \leftarrow i + 1$;
      end while;
      $R \leftarrow R \setminus \{s\}$; $u \leftarrow i$;
      while $i \leq |w|$ and $\delta(s, w_i) \neq \bot$ do
         $\delta(s, w_i) \leftarrow$ clone$(\delta(s, w_i))$; $s \leftarrow \delta(s, w_i)$;
         push$(s, P)$; $i \leftarrow i + 1$;
      end while;
      while $i \leq |w|$ do
         $s \leftarrow$ new state; $s \leftarrow \delta(s, w_i)$;
         push$(s, P)$; $i \leftarrow i + 1$;
      end while;
      $F \leftarrow F \cup \{s\}$; pop$(P)$;
      while $P$ not empty do
         if $\exists_{r \in R}\, r \equiv s$ then
            if $i = u$ and $i > 0$ then
               $R \leftarrow R \setminus \{$top$(P)\}$; $u \leftarrow u - 1$;
            end if;
            delete $s$; $\delta($top$(P), w_i) \leftarrow r$;
         else
            $R \leftarrow R \cup \{s\}$;
            if $i = u$ then break; end if;
         end if;
         $i \leftarrow i - 1$; $s \leftarrow$ pop$(P)$;
      end while;
      reset $P$;
   end while;
end function;
```



peux

Complexity and performance

- Both algorithms keep intermediate automata (almost) minimal
- Both run in time proportional to input data size
- The algorithm for sorted data is faster but less flexible
- Traditional algorithms are slower and use much more memory
- There are extensions of both algorithms to the case off adding words to a cyclic automaton